Chapter 13: Oscillations

Comprehensive Study Notes

Class 11 Physics - NCERT Based

EXAM SPRINT - Complete Coverage for JEE and NEET Examinations

13.1 INTRODUCTION

What is Oscillatory Motion?

Definition: Motion that repeats itself about a mean position in a to-and-fro manner.

Key Characteristics:

- Repetitive nature
- Motion about equilibrium position
- Net restoring force towards mean position
- Different from general periodic motion

Examples of Oscillatory Motion:

- Pendulum of wall clock
- Vibrating strings in musical instruments
- Boat tossing in water
- Piston in steam engine
- AC voltage variations
- Atomic vibrations in solids

Importance in Physics:

- Foundation for understanding wave phenomena
- Essential for musical instrument design
- Critical in AC circuit analysis
- Basis for understanding molecular vibrations
- Temperature-energy relationship in solids

13.2 PERIODIC AND OSCILLATORY MOTIONS

Periodic Motion

Definition: Motion that repeats itself at regular intervals of time.

Mathematical Condition:

$$f(t) = f(t + T) \text{ for all } t$$

where T is the period.

Period and Frequency

Period (T):

- Smallest time interval after which motion repeats
- SI Unit: second (s)
- Special units: microseconds (μs) for fast vibrations

Frequency (ν):

$$\nu = 1/T$$

- Number of oscillations per unit time
- SI Unit: hertz (Hz) = s^{-1}
- Named after Heinrich Hertz

Example Calculation: Human heart beats 75 times per minute:

- Frequency = 75/60 = 1.25 Hz
- Period = 1/1.25 = 0.8 s

Displacement in Oscillations

General Definition: Change with time of any physical property under consideration.

Types of Displacement Variables:

- Position displacement (spring-mass system)
- Angular displacement (pendulum)
- Voltage variations (AC circuits)
- Pressure variations (sound waves)
- Electric/magnetic field variations (light waves)

Mathematical Representation: Simple periodic functions:

$$f(t) = A \cos \omega t$$
$$f(t) = A \sin \omega t$$
$$f(t) = A \sin \omega t + B \cos \omega t$$

Period Relationship:

$$T = 2\pi/\omega$$

Fourier's Theorem

Statement: Any periodic function can be expressed as a superposition of sine and cosine functions with suitable coefficients.

Mathematical Form:

$$f(t) = A \sin \omega t + B \cos \omega t = D \sin(\omega t + \varphi)$$

where:

$$D = \sqrt{(A^2 + B^2)}$$
$$\tan \varphi = B/A$$

13.3 SIMPLE HARMONIC MOTION (SHM)

Definition

Mathematical Expression:

$$x(t) = A \cos(\omega t + \varphi)$$

Key Parameters:

- **A:** Amplitude (maximum displacement)
- **ω:** Angular frequency (rad/s)
- **($\omega t + \varphi$):** Phase of motion
- **φ:** Phase constant/initial phase

Characteristics of SHM

Amplitude (A):

- Magnitude of maximum displacement
- Always positive by convention
- Determines energy of oscillation
- Independent of ω and φ

Angular Frequency (ω):

$$\omega = 2\pi/T = 2\pi\nu$$

- Rate of change of phase
- SI Unit: rad/s
- Related to period and frequency

Phase ($\omega t + \varphi$):

- Determines state of motion at time t
- Complete description requires position and velocity
- Advances by 2π in one complete cycle

Phase Constant (φ):

- Phase at $t = 0$
- Determined by initial conditions
- Can be determined from $x(0)$ if A is known

Types of Motion Analysis

Example Functions:

1. **$\sin \omega t + \cos \omega t$**

   $= \sqrt{2} \sin(\omega t + \pi/4)$

   - Simple harmonic with period $2\pi/\omega$
   - Phase constant $\pi/4$

2. **$\sin^2 \omega t$**

   $= 1/2 - 1/2 \cos 2\omega t$

   - Periodic with period π/ω
   - Not simple harmonic (equilibrium at 1/2)

3. **$e^{-}\omega t$**

   - Non-periodic (monotonically decreasing)
   - Cannot represent physical displacement

13.4 SHM AND UNIFORM CIRCULAR MOTION

Geometrical Connection

Key Insight: SHM is the projection of uniform circular motion on any diameter.

Mathematical Derivation: Consider particle P moving on circle of radius A with angular speed ω:

- Position vector makes angle $(\omega t + \varphi)$ with x-axis
- x-projection: $x(t) = A \cos(\omega t + \varphi)$
- y-projection: $y(t) = A \sin(\omega t + \varphi)$

Reference Circle Method:

- Reference particle: P (moving on circle)

- Reference circle: Circle of radius A
- Projection particle: P' (on diameter)
- Both projections are SHM with phase difference $\pi/2$

Important Note: Forces in linear SHM ≠ centripetal force in circular motion

Example Applications

Case 1: Anticlockwise motion

- Initial angle: $\pi/4$
- Period: T = 4s
- $x(t) = A \cos(\pi t/2 + \pi/4)$

Case 2: Clockwise motion

- Initial angle: $\pi/2$
- Period: T = 30s
- $x(t) = B \sin(2\pi t/30) = B \cos(2\pi t/30 - \pi/2)$

13.5 VELOCITY AND ACCELERATION IN SHM

Velocity in SHM

Mathematical Derivation:

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x(t) = A cos(ωt + φ)
v(t) = dx/dt = -ωA sin(ωt + φ)
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Key Properties:

- Maximum velocity: $v_{max} = \omega A$ (at mean position)

- Zero velocity at extreme positions ($x = \pm A$)
- Phase difference: $\pi/2$ ahead of displacement
- Velocity amplitude: ωA

Acceleration in SHM

Mathematical Derivation:

$$v(t) = -\omega A \sin(\omega t + \varphi)$$
$$a(t) = dv/dt = -\omega^2 A \cos(\omega t + \varphi) = -\omega^2 x(t)$$

Key Properties:

- Maximum acceleration: $a_{max} = \omega^2 A$ (at extreme positions)
- Zero acceleration at mean position
- Phase difference: π ahead of displacement
- Always directed toward mean position
- Acceleration amplitude: $\omega^2 A$

Phase Relationships

For $\varphi = 0$:
$$x(t) = A \cos \omega t$$
$$v(t) = -\omega A \sin \omega t = \omega A \cos(\omega t + \pi/2)$$
$$a(t) = -\omega^2 A \cos \omega t = \omega^2 A \cos(\omega t + \pi)$$

Phase Differences:

- v leads x by $\pi/2$
- a leads x by π

- a leads v by $\pi/2$

Numerical Example

Given: $x = 5 \cos(2\pi t + \pi/4)$, $t = 1.5$ s

Solutions:

- Displacement: $x = 5 \cos(3\pi + \pi/4) = -3.535$ m
- Speed: $|v| = 5 \times 2\pi \times |\sin(3\pi + \pi/4)| = 22$ m/s
- Acceleration: $a = -(2\pi)^2 \times (-3.535) = 140$ m/s^2

13.6 FORCE LAW FOR SHM

Newton's Second Law Application

$$F(t) = ma(t) = -m\omega^2 x(t)$$

Hooke's Law Form:

$$F(t) = -kx(t)$$

where $k = m\omega^2$ (spring constant)

Key Relationships

$$\omega = \sqrt{(k/m)}$$
$$T = 2\pi\sqrt{(m/k)}$$

Restoring Force Characteristics

- Proportional to displacement

- Always directed toward equilibrium
- Linear relationship (linear harmonic oscillator)
- Conservative force

Spring Systems Analysis

Two Springs in Parallel:

- Each spring: force = -kx
- Total force: F = -2kx
- Effective spring constant: $k_eff = 2k$
- Period: $T = 2\pi\sqrt{(m/2k)}$

Non-linear Oscillators: Real systems may have additional terms:

- $F = -kx - \alpha x^2 - \beta x^3 + ...$
- More complex motion patterns
- Anharmonic oscillations

13.7 ENERGY IN SHM

Kinetic Energy

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \varphi)$$
$$K = \frac{1}{2}kA^2\sin^2(\omega t + \varphi)$$

Properties:

- Maximum at mean position: $K_{max} = \frac{1}{2}kA^2$
- Zero at extreme positions

- Period of oscillation: T/2
- Always positive

Potential Energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$$

Properties:

- Maximum at extreme positions: $U_{max} = \frac{1}{2}kA^2$
- Zero at mean position
- Period of oscillation: T/2
- Always positive

Total Energy

$$E = K + U = \frac{1}{2}kA^2[\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi)]$$
$$E = \frac{1}{2}kA^2 = \text{constant}$$

Energy Conservation:

- Total mechanical energy constant
- Energy oscillates between kinetic and potential
- Both K and U oscillate with period T/2
- Maximum energy equals initial potential or kinetic energy

Energy Transformation

- At mean position: E = K (all kinetic)
- At extremes: E = U (all potential)

- Continuous energy exchange during motion
- No energy loss in ideal SHM (no friction)

Numerical Example

Given: m = 1 kg, k = 50 N/m, A = 10 cm

At x = 5 cm:

- K.E. = $\frac{1}{2}m(0.61)^2 = 0.19$ J
- P.E. = $\frac{1}{2}k(0.05)^2 = 0.0625$ J
- Total Energy = 0.25 J

Verification:

- At maximum displacement: $E = \frac{1}{2}k(0.1)^2 = 0.25$ J ✓

13.8 THE SIMPLE PENDULUM

Historical Context

Galileo's Observations:

- Chandelier oscillations in church
- Periodic nature independent of amplitude (for small angles)
- Foundation for pendulum clocks

Physical Setup

Components:

- Small bob of mass m
- Inextensible, massless string of length L

- Fixed support point
- Motion in vertical plane

Force Analysis

Forces on Bob:

- Tension T (along string)
- Weight mg (vertically downward)

Force Components:

- Radial: $T - mg \cos \theta$ (provides centripetal acceleration)
- Tangential: $-mg \sin \theta$ (provides restoring torque)

Torque Equation

$$\tau = -L(mg \sin \theta)$$

Using $\tau = I\alpha$:

$$I\alpha = -mgL \sin \theta$$

For small angles (θ in radians):

$$\sin \theta \approx \theta$$

Simplified Equation:

$$\alpha = -(mgL/I)\theta$$

For simple pendulum: $I = mL^2$

$$\alpha = -(g/L)\theta$$

Time Period Derivation

Comparing with SHM:

$$\alpha = -\omega^2\theta$$

Therefore:

$$\omega^2 = g/L$$
$$\omega = \sqrt{(g/L)}$$

Period Formula:

$$T = 2\pi\sqrt{(L/g)}$$

Small Angle Approximation

Validity:

θ (degrees)	θ (radians)	sin θ	Error %
5°	0.087	0.087	0.0%
10°	0.175	0.174	0.5%
15°	0.262	0.259	1.2%
20°	0.349	0.342	2.0%

Conclusion: Approximation valid for $\theta \leq 15°$

Applications

Example: Seconds pendulum ($T = 2s$)

$$L = gT^2/4\pi^2 = 9.8 \times 4/4\pi^2 = 1 \text{ m}$$

SUMMARY - KEY CONCEPTS

1. Motion Classifications

- **Periodic Motion:** Repeats after time T
- **Oscillatory Motion:** To-and-fro about mean position
- **Simple Harmonic Motion:** Sinusoidal displacement function

2. SHM Definitions

Displacement: $x(t) = A\cos(\omega t + \varphi)$ **Velocity:** $v(t) = -\omega A\sin(\omega t + \varphi)$ **Acceleration:** $a(t) = -\omega^2 A \cos(\omega t + \varphi) = -\omega^2 x$

3. Force Law

$$F = -kx = -m\omega^2 x$$

4. Energy Relations

- **Kinetic:** $K = \frac{1}{2}mv^2 = \frac{1}{2}kA^2\sin^2(\omega t + \varphi)$
- **Potential:** $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$
- **Total:** $E = K + U = \frac{1}{2}kA^2 = \text{constant}$

5. Simple Pendulum

$T = 2\pi\sqrt{(L/g)}$ (for small angles)

POINTS TO PONDER - CRITICAL INSIGHTS

1. Period Independence

- SHM period independent of amplitude or energy
- Only depends on system parameters (m, k, L, g)
- Contrast with planetary motion (Kepler's third law)

2. Phase Relationships

- Velocity leads displacement by $\pi/2$
- Acceleration leads displacement by π
- Both kinetic and potential energy oscillate with period $T/2$

3. Reference Circle Connection

- SHM as projection of circular motion
- Different force systems despite mathematical similarity
- Useful for visualization and problem-solving

4. Energy Conservation

- Total mechanical energy constant in ideal SHM
- Continuous transformation between K.E. and P.E.
- Energy proportional to amplitude squared

5. Small Angle Approximation

- Critical for simple pendulum analysis
- $\sin\theta \approx \theta$ valid for $\theta \leq 15°$
- Larger angles lead to non-harmonic motion

6. Initial Conditions

- Two initial conditions determine SHM completely
- Common sets: (x_0, v_0), (A, φ), (E, φ)
- Phase constant determined by initial state

JEE/NEET SPECIFIC IMPORTANT POINTS

High-Yield Topics

1. SHM Equations and Relationships

Master these formulas:

$$x = A\cos(\omega t + \varphi)$$
$$v = -\omega A\sin(\omega t + \varphi)$$
$$a = -\omega^2 A\cos(\omega t + \varphi)$$
$$\omega = \sqrt{(k/m)} = 2\pi/T$$
$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}m\omega^2 A^2$$

2. Spring-Mass Systems

Key scenarios:

- Single spring vertical/horizontal
- Two springs in series/parallel
- Variable mass problems

- Energy analysis

3. Simple Pendulum

Standard problems:

- Period calculation
- Length determination
- Gravity variation effects
- Small angle validity

4. Energy Analysis

Critical concepts:

- Energy conservation
- K.E. and P.E. variations
- Maximum speed and acceleration points
- Energy-amplitude relationships

Common Question Types

1. Direct Application Problems

- Given amplitude, frequency → find displacement at time t
- Given spring constant, mass → find period
- Energy calculations at different positions

2. Graphical Analysis

- Sketch x-t, v-t, a-t graphs

- Interpret phase relationships
- Calculate areas and slopes

3. System Analysis

- Multiple spring arrangements
- Pendulum with varying parameters
- Energy transformation problems

4. Comparative Studies

- SHM vs circular motion
- Different oscillating systems
- Effect of parameter changes

Problem-Solving Strategy

1. Identification Phase

- Recognize SHM from given conditions
- Identify system parameters (m, k, L, g)
- Determine initial conditions

2. Setup Phase

- Choose appropriate SHM equation
- Apply initial conditions to find A and φ
- Set up coordinate system consistently

3. Calculation Phase

- Substitute known values
- Use appropriate relationships
- Check dimensional consistency

4. Verification Phase

- Verify physical reasonableness
- Check limiting cases
- Ensure energy conservation

MEMORY AIDS AND MNEMONICS

SHM Equations

"X-V-A Sequence"

- $X = A \cos(\omega t + \varphi)$
- $V = -\omega A \sin(\omega t + \varphi)$ [derivative of X]
- $A = -\omega^2 A \cos(\omega t + \varphi)$ [derivative of V]

Phase Relationships

"V-A Lead X"

- Velocity leads displacement by $\pi/2$
- Acceleration leads displacement by π
- Acceleration leads velocity by $\pi/2$

Energy Relations

"K-U Dance"

- When K is max, U is min (at mean position)
- When U is max, K is min (at extremes)
- Total energy = max K = max U

Pendulum Formula

"Length over Gravity"

```
T = 2π√(L/g)
```

Remember: Period increases with length, decreases with gravity

PRACTICE PROBLEMS FOR JEE/NEET

Level 1: Basic Application

Problem 1: A particle executes SHM with amplitude 5 cm and period 4 s. Find: (a) Maximum velocity (b) Maximum acceleration

Solution:

- $\omega = 2\pi/T = \pi/2$ rad/s
- $v_{max} = \omega A = (\pi/2) \times 0.05 = 0.0785$ m/s
- $a_{max} = \omega^2 A = (\pi^2/4) \times 0.05 = 0.123$ m/s^2

Problem 2: A spring-mass system has k = 100 N/m and m = 0.5 kg. Find the period of oscillation.

Solution:

```
T = 2π√(m/k) = 2π√(0.5/100) = 2π√(0.005) = 0.445 s
```

Level 2: Intermediate

Problem 3: A simple pendulum has period 2 s on earth. What will be its period on moon where g = 1.6 m/s²?

Solution:

- On earth: $T_1 = 2\pi\sqrt{(L/g_1)}$
- On moon: $T_2 = 2\pi\sqrt{(L/g_2)}$
- $T_2/T_1 = \sqrt{(g_1/g_2)} = \sqrt{(9.8/1.6)} = 2.47$
- $T_2 = 2 \times 2.47 = 4.94$ s

Problem 4: In SHM, when displacement is half the amplitude, find the ratio of kinetic to potential energy.

Solution:

- At $x = A/2$: $U = \frac{1}{2}k(A/2)^2 = \frac{1}{8}kA^2$
- Total energy: $E = \frac{1}{2}kA^2$
- $K = E - U = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = \frac{3}{8}kA^2$
- $K/U = (\frac{3}{8}kA^2)/(\frac{1}{8}kA^2) = 3$

Level 3: Advanced

Problem 5: Two identical springs (k each) are connected to a mass m: (a) in series (b) in parallel. Compare the periods.

Solution:

- Series: $k_eff = k/2 \rightarrow T_1 = 2\pi\sqrt{(2m/k)}$
- Parallel: $k_eff = 2k \rightarrow T_2 = 2\pi\sqrt{(m/2k)}$

- $T_1/T_2 = \sqrt{(2m/k)} \times \sqrt{(2k/m)} = 2$

ADVANCED TOPICS FOR JEE

1. Damped Oscillations

- Exponential decay of amplitude
- Quality factor and damping coefficient
- Critical, under, and over damping

2. Forced Oscillations

- External periodic driving force
- Resonance phenomena
- Amplitude and phase response

3. Coupled Oscillations

- Two-mass spring systems
- Normal modes of vibration
- Beat phenomena

4. Non-linear Oscillations

- Anharmonic terms in restoring force
- Period dependence on amplitude
- Approximate solution methods

ERROR ANALYSIS IN OSCILLATIONS

Common Mistakes

1. Phase Confusion

- Wrong initial phase determination
- Incorrect phase relationships
- Sign errors in trigonometric functions

2. Energy Calculations

- Forgetting factor of ½ in energy expressions
- Wrong energy conservation applications
- Mixing kinetic and potential energy concepts

3. Period Formulas

- Confusing different system formulas
- Wrong parameter identification
- Unit conversion errors

Prevention Strategies

1. Always draw clear diagrams with coordinate systems
2. List all given parameters systematically
3. Check energy conservation at key points
4. Verify dimensional consistency of all equations

EXPERIMENTAL CONNECTIONS

1. Historical Experiments

- Galileo's pendulum observations
- Hooke's spring law investigations
- Development of mechanical clocks

2. Modern Applications

- Seismic monitoring systems
- Precision timing devices
- Structural vibration analysis

3. Laboratory Techniques

- Digital oscilloscopes for SHM visualization
- Motion sensors for real-time data
- Computer modeling of oscillatory systems

EXAM SPRINT - FINAL CHECKLIST

Essential Formulas to Remember

$x = A \cos(\omega t + \varphi)$
$v = -\omega A \sin(\omega t + \varphi)$
$a = -\omega^2 x$
$\omega = \sqrt{(k/m)} = 2\pi/T$
$E = \frac{1}{2}kA^2$
$T_{pendulum} = 2\pi\sqrt{(L/g)}$

Key Concepts to Master

1. SHM definition and characteristics
2. Energy conservation in oscillations
3. Phase relationships between x, v, a
4. Simple pendulum motion
5. Spring-mass system analysis

Problem-Solving Priorities

1. Identify SHM conditions
2. Apply initial conditions correctly
3. Use energy conservation
4. Verify answers physically

EXAM SPRINT - Master Oscillations through focused study of SHM equations, energy analysis, and pendulum motion. Regular practice of numerical problems and conceptual understanding is essential for JEE/NEET success.

Source: NCERT Physics Class 11, Chapter 13 - Comprehensive coverage for competitive exam preparation