Chapter 3: Motion in a Plane

Comprehensive Study Notes

Class 11 Physics - NCERT Based

EXAM SPRINT - Complete Coverage for JEE and NEET Examinations

3.1 INTRODUCTION

What is Motion in a Plane?

Definition: Motion of objects in two dimensions, requiring vector analysis for complete description.

Key Characteristics:

- Extension of one-dimensional motion concepts
- Requires vector quantities for position, displacement, velocity, acceleration
- Directional aspects cannot be handled by simple +/- signs
- Foundation for projectile motion and circular motion

Examples of Plane Motion:

- Projectile motion (ball thrown at angle)
- Circular motion (satellite orbits)
- Vehicle motion on curved paths
- Sports ball trajectories

Motion Analysis Framework

Two-Dimensional Approach:

- Motion treated as combination of two perpendicular one-dimensional motions
- Independent analysis along x and y axes
- Vector addition for resultant quantities

3.2 SCALARS AND VECTORS

Scalar Quantities

Definition: Quantities with magnitude only **Examples**: Distance, speed, mass, temperature, time, energy **Mathematical Operations**: Follow ordinary algebra rules

- Addition: 5 m + 3 m = 8 m
- Multiplication: 2 kg × 3 = 6 kg

Vector Quantities

Definition: Quantities with both magnitude and direction, obeying triangle/parallelogram law of addition **Examples**: Displacement, velocity, acceleration, force **Notation**:

- Bold face: **v**
- Arrow notation: v⃗ (handwritten) **Magnitude**: |**v**| = v

Position and Displacement Vectors

Position Vector:

- **r** = OP⃗ from origin O to point P
- Components: **r** = x**î** + y**ĵ**
- Magnitude: |**r**| = $\sqrt{x^2 + y^2}$

Displacement Vector:

- Δ**r** = **r**' - **r** (straight line from initial to final position)

- **Key Property**: Independent of actual path taken
- Always points from initial to final position

Equality of Vectors

Condition: Two vectors equal if and only if:

1. Same magnitude
2. Same direction **Mathematical**: **A** = **B** ⟺ |**A**| = |**B**| and direction(A) = direction(B)

3.3 MULTIPLICATION OF VECTORS BY REAL NUMBERS

Scalar Multiplication Rules

Positive Scalar λ > 0:

- λ**A** has magnitude λ|**A**|
- Same direction as **A**
- Example: 3**A** is three times longer, same direction

Negative Scalar -λ < 0:

- |-λ**A**| = λ|**A**|
- Opposite direction to **A**
- Example: -2**A** is twice as long, opposite direction

Physical Interpretation:

- If λ has dimensions: [λ**A**] = [λ] × [**A**]
- Example: velocity × time = displacement

3.4 ADDITION AND SUBTRACTION OF VECTORS

Graphical Methods

Triangle Method (Head-to-Tail):

1. Place tail of second vector at head of first
2. Resultant connects tail of first to head of second
3. **R** = **A** + **B**

Parallelogram Method:

1. Place both vectors at common origin
2. Complete parallelogram
3. Diagonal represents resultant

Vector Addition Laws

Commutative Law: **A** + **B** = **B** + **A** **Associative Law**: (**A** + **B**) + **C** = **A** + (**B** + **C**)

Null Vector

Definition: Vector with zero magnitude **Properties**:

- **A** + **0** = **A**
- λ**0** = **0**
- **0A** = **0**
- Direction undefined due to zero magnitude

Vector Subtraction

Definition: **A** - **B** = **A** + (-**B**) **Method**: Add negative of second vector to first

3.5 RESOLUTION OF VECTORS

Component Resolution

Process: Express vector as sum of component vectors along chosen directions **Mathematical**: **A** = λ**a** + μ**b** (where **a**, **b** are reference vectors)

Unit Vectors

Definition: Vector of unit magnitude pointing in specific direction **Standard Unit Vectors**:

- $\hat{\mathbf{i}}$: along positive x-axis
- $\hat{\mathbf{j}}$: along positive y-axis
- $\hat{\mathbf{k}}$: along positive z-axis **Properties**: $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$

Rectangular Components

Two Dimensions:

- $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$
- $A_x = A \cos \theta$ (x-component)
- $A_y = A \sin \theta$ (y-component)
- $|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2)}$
- $\tan \theta = A_y/A_x$

Three Dimensions:

- $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_i\hat{\mathbf{k}}$
- $|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2 + A_i^2)}$

3.6 VECTOR ADDITION - ANALYTICAL METHOD

Component Addition

For vectors: $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}$ **Resultant**: $\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}}$

Component Rules:

- $R_x = A_x + B_x$
- $R_y = A_y + B_y$
- $|\mathbf{R}| = \sqrt{(R_x^2 + R_y^2)}$

Law of Cosines and Sines

For two vectors A, B with angle θ between them:

Magnitude: $|\mathbf{R}|^2 = A^2 + B^2 + 2AB \cos \theta$

Direction: $\sin \alpha/B = \sin \theta/|\mathbf{R}|$ (where α is angle R makes with A)

3.7 MOTION IN A PLANE

Position Vector and Displacement

Position: $\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$ **Displacement**: $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = \Delta x\hat{\mathbf{i}} + \Delta y\hat{\mathbf{j}}$

Velocity in Two Dimensions

Average Velocity: $\bar{\mathbf{v}} = \Delta\mathbf{r}/\Delta t = (\Delta x/\Delta t)\hat{\mathbf{i}} + (\Delta y/\Delta t)\hat{\mathbf{j}}$

Instantaneous Velocity: $\mathbf{v} = \lim(\Delta t\to 0)\ \Delta\mathbf{r}/\Delta t = d\mathbf{r}/dt$ $\mathbf{v} = (dx/dt)\hat{\mathbf{i}} + (dy/dt)\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

Properties:

- Always tangent to path

- Magnitude: $|\mathbf{v}| = \sqrt{(v_x^2 + v_y^2)}$
- Direction: $\tan \theta = v_y/v_x$

Acceleration in Two Dimensions

Average Acceleration: $\bar{\mathbf{a}} = \Delta\mathbf{v}/\Delta t = (\Delta v_x/\Delta t)\hat{\mathbf{i}} + (\Delta v_y/\Delta t)\hat{\mathbf{j}}$

Instantaneous Acceleration: $\mathbf{a} = \lim(\Delta t \to 0)\ \Delta\mathbf{v}/\Delta t = d\mathbf{v}/dt$ $\mathbf{a} = (dv_x/dt)\hat{\mathbf{i}} + (dv_y/dt)\hat{\mathbf{j}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$

Key Point: Direction of **a** can be at any angle to **v** (0° to 180°)

3.8 MOTION IN A PLANE WITH CONSTANT ACCELERATION

Kinematic Equations for 2D Motion

For constant acceleration a = $a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$:

Velocity Equations:

- $v_x = v_{0x} + a_x t$
- $v_y = v_{0y} + a_y t$
- $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$

Position Equations:

- $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$
- $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$
- $\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}t + \frac{1}{2}\mathbf{a}t^2$

Key Principle: Motion in perpendicular directions can be treated independently

Example Analysis

Problem: Particle with $\mathbf{v_0} = 5\hat{\mathbf{i}}$ m/s, $\mathbf{a} = (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m/s^2

Solution Process:

1. Identify components: $v_{0x} = 5$ m/s, $v_{0y} = 0$, $a_x = 3$ m/s², $a_y = 2$ m/s²
2. Apply equations: $x = 5t + 1.5t^2$, $y = t^2$
3. Find velocity: $\mathbf{v} = (5 + 3t)\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}$

3.9 PROJECTILE MOTION

Basic Setup

Definition: Motion of object under gravity alone (neglecting air resistance)

Initial Conditions:

- Launch velocity: $\mathbf{v_0} = v_0 \cos \theta_0\hat{\mathbf{i}} + v_0 \sin \theta_0\hat{\mathbf{j}}$
- Launch angle: θ_0 with horizontal
- Acceleration: $\mathbf{a} = -g\hat{\mathbf{j}}$ (downward)

Component Analysis

Horizontal Motion ($a_x = 0$):

- $v_x = v_0 \cos \theta_0$ (constant)
- $x = (v_0 \cos \theta_0)t$

Vertical Motion ($a_y = -g$):

- $v_y = v_0 \sin \theta_0 - gt$
- $y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$

Trajectory Equation

Path Shape: Parabola **Equation**: $y = x \tan \theta_0 - (gx^2)/(2v_0^2 \cos^2 \theta_0)$

Key Results

Time of Flight: $T_F = (2v_0 \sin \theta_0)/g$

Maximum Height: $h_m = (v_0^2 \sin^2 \theta_0)/(2g)$

Range: $R = (v_0^2 \sin 2\theta_0)/g$

Maximum Range: Occurs at $\theta_0 = 45°$, $R_{max} = v_0^2/g$

Important Properties

1. **Symmetry**: Time to reach max height = time to fall from max height
2. **Equal ranges**: For angles θ and (90° - θ)
3. **Independence**: Horizontal and vertical motions are independent

Solved Examples

Example 1: Ball thrown at 28 m/s, 30° above horizontal

- Maximum height: $h_m = (28^2 \sin^2 30°)/(2\times9.8) = 10.0$ m
- Time of flight: $T_F = (2\times28\times\sin 30°)/9.8 = 2.9$ s
- Range: $R = (28^2 \sin 60°)/9.8 = 69$ m

Example 2: Stone thrown horizontally from 490 m height at 15 m/s

- Time to hit ground: $490 = \frac{1}{2}gt^2 \rightarrow t = 10$ s
- Final velocity: $v_x = 15$ m/s, $v_y = -98$ m/s
- Speed on impact: $|\mathbf{v}| = \sqrt{(15^2 + 98^2)} = 99$ m/s

3.10 UNIFORM CIRCULAR MOTION

Definition and Characteristics

Uniform Circular Motion: Object moving in circle with constant speed

- Speed constant, velocity direction continuously changing
- Always experiences acceleration toward center

Centripetal Acceleration

Derivation: Using limiting process as $\Delta t \to 0$ **Magnitude**: $a^c = v^2/R$ **Direction**: Always toward center of circle **Alternative Form**: $a^c = \omega^2 R$

Angular Quantities

Angular Displacement: θ (radians) **Angular Speed**: $\omega = d\theta/dt = 2\pi/T = 2\pi\nu$ **Relationships**:

- $v = \omega R$ (linear speed)
- $T = 2\pi/\omega$ (period)
- $\nu = 1/T$ (frequency)

Acceleration Analysis

Key Points:

1. Magnitude of acceleration constant: $|\mathbf{a}| = v^2/R$
2. Direction continuously changing (always toward center)
3. **a** is NOT a constant vector (direction changes)
4. $\mathbf{a} \perp \mathbf{v}$ always

Example Problem

Insect in circular groove: $R = 12$ cm, 7 revolutions in 100 s

- Angular speed: $\omega = 2\pi \times 7/100 = 0.44$ rad/s
- Linear speed: $v = \omega R = 5.3$ cm/s
- Acceleration magnitude: $a = \omega^2 R = 2.3$ cm/s^2

SUMMARY - KEY CONCEPTS

1. Vector Fundamentals

- **Scalars**: Magnitude only (distance, speed, mass)
- **Vectors**: Magnitude + direction (displacement, velocity, acceleration)
- **Operations**: Follow specific vector algebra rules

2. Vector Representation

- **Unit vectors**: $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ for coordinate directions
- **Components**: $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_i\hat{\mathbf{k}}$
- **Magnitude**: $|\mathbf{A}| = \sqrt{}(A_x^2 + A_y^2 + A_i^2)$

3. Motion Analysis

- **Position**: $\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$
- **Velocity**: $\mathbf{v} = d\mathbf{r}/dt$ (tangent to path)
- **Acceleration**: $\mathbf{a} = d\mathbf{v}/dt$

4. Kinematic Equations (Constant Acceleration)

- $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$

- $\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}t + \frac{1}{2}\mathbf{a}t^2$
- Component-wise application

5. Projectile Motion

- **Horizontal**: Uniform motion ($a_x = 0$)
- **Vertical**: Uniformly accelerated motion ($a_y = -g$)
- **Trajectory**: Parabolic path
- **Key formulas**: Range, maximum height, time of flight

6. Circular Motion

- **Centripetal acceleration**: $a^c = v^2/R$ toward center
- **Angular speed**: $\omega = v/R = 2\pi/T$
- **Always accelerated** (direction of **v** changing)

POINTS TO PONDER - CRITICAL INSIGHTS

1. Vector vs Scalar Distinction

- Vectors need both magnitude and direction specification
- Vector addition ≠ algebraic addition
- Direction matters in all vector operations

2. Independence Principle

- Perpendicular components of motion are independent
- Horizontal motion unaffected by vertical motion in projectile motion
- Allows separate analysis of x and y components

3. Acceleration Misconceptions

- **Zero velocity ≠ zero acceleration** (ball at max height)
- Acceleration can be perpendicular to velocity (circular motion)
- Constant speed ≠ constant velocity (direction can change)

4. Circular Motion Insights

- **Uniform** refers to constant speed, not constant velocity
- Always accelerated motion due to direction change
- Centripetal acceleration is not constant vector (magnitude constant, direction changing)

5. Projectile Motion Key Points

- **Parabolic trajectory** for all projectile motion
- **45° gives maximum range** for given launch speed
- **Complementary angles** give equal ranges
- **Time of flight** depends only on vertical motion

JEE/NEET SPECIFIC IMPORTANT POINTS

High-Yield Topics

1. Vector Operations

Master these skills:

- Component resolution: $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$
- Vector addition: $\mathbf{R} = \mathbf{A} + \mathbf{B}$
- Magnitude calculation: $|\mathbf{R}| = \sqrt{(R_x^2 + R_y^2)}$
- Direction finding: $\tan \theta = R_y/R_x$

2. Projectile Motion Analysis

Standard scenarios:

- Object projected at angle θ
- Object dropped from height
- Object thrown horizontally
- Maximum range problems

Key formulas to memorize:

- Range: $R = (v_0^2 \sin 2\theta)/g$
- Max height: $h_m = (v_0^2 \sin^2 \theta)/(2g)$
- Time of flight: $T = (2v_0 \sin \theta)/g$

3. Circular Motion

Applications:

- Centripetal acceleration: $a^c = v^2/R = \omega^2 R$
- Banking of roads
- Vertical circular motion
- Satellite motion basics

4. Relative Motion in 2D

Vector relationships:

- $\mathbf{v}_{12} = \mathbf{v}_1 - \mathbf{v}_2$
- Rain-umbrella problems
- River crossing problems

Common Question Types

1. Direct Application Problems

- Vector addition/subtraction
- Component resolution
- Projectile motion calculations
- Circular motion parameters

2. Graphical Problems

- Trajectory sketching
- Component vs time graphs
- Vector diagram construction

3. Multi-Stage Problems

- Projectile motion with obstacles
- Combined linear and circular motion
- Motion in different reference frames

4. Real-World Applications

- Sports ball trajectories
- Vehicle motion on curved roads
- Satellite and planetary motion

PROBLEM-SOLVING STRATEGY

1. Setup Phase

- **Choose coordinate system** (origin and axes)
- **Identify vector quantities** (position, velocity, acceleration)
- **Resolve into components** if needed
- **List given and required quantities**

2. Analysis Phase

- **Apply appropriate equations**:
  - Constant acceleration: kinematic equations
  - Circular motion: centripetal acceleration
  - Projectile motion: separate x,y analysis
- **Work with components independently**
- **Maintain sign conventions**

3. Calculation Phase

- **Substitute values with correct signs**
- **Solve algebraically first**, then numerically
- **Check dimensional consistency**
- **Calculate magnitude and direction for vectors**

4. Verification Phase

- **Physical reasonableness check**
- **Verify signs and magnitudes**

- **Check limiting cases**
- **Units consistency**

MEMORY AIDS AND MNEMONICS

Vector Components

"SOH-CAH-TOA"

- $A_x = A \cos \theta$ (Adjacent/Hypotenuse)
- $A_y = A \sin \theta$ (Opposite/Hypotenuse)

Projectile Motion

"Rise-Run-Range"

- Rise: $h_m = (v_0^2 \sin^2 \theta)/(2g)$
- Run: $T = (2v_0 \sin \theta)/g$
- Range: $R = (v_0^2 \sin 2\theta)/g$

Circular Motion

"Speed-Round-Center"

- Speed: $v = \omega R$
- Round: $T = 2\pi/\omega$
- Center: $a^c = v^2/R$

PRACTICE PROBLEMS FOR JEE/NEET

Level 1: Basic Application

Problem 1: Vector $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$, $\mathbf{B} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$. Find $\mathbf{A} + \mathbf{B}$ and $|\mathbf{A} + \mathbf{B}|$.

Solution:

- **A** + **B** = (3+2)**î** + (4-3)**ĵ** = 5**î** + **ĵ**
- |**A** + **B**| = $\sqrt{(5^2 + 1^2)} = \sqrt{26}$

Problem 2: Projectile launched at 20 m/s, 60° above horizontal. Find maximum height.

Solution:

- $h_m = (v_0^2 \sin^2 \theta)/(2g) = (20^2 \times \sin^2 60°)/(2\times10) = (400 \times 0.75)/20 = 15$ m

Level 2: Intermediate

Problem 3: Rain falls vertically at 10 m/s. Wind blows horizontally at 6 m/s. At what angle should umbrella be held?

Solution:

- Resultant velocity makes angle θ with vertical
- $\tan \theta = 6/10 = 0.6$
- $\theta = \tan^{-1}(0.6) \approx 31°$

Problem 4: Particle in uniform circular motion, radius 2 m, period 4 s. Find centripetal acceleration.

Solution:

- $\omega = 2\pi/T = 2\pi/4 = \pi/2$ rad/s
- $a^c = \omega^2 R = (\pi/2)^2 \times 2 = \pi^2/2 \approx 4.9$ m/s^2

Level 3: Advanced

Problem 5: Projectile launched from 100 m height horizontally at 30 m/s. Find range and impact velocity.

Solution:

- Time to hit ground: $100 = \frac{1}{2}gt^2 \rightarrow t = \sqrt{(200/10)} = \sqrt{20}$ s
- Range: $R = v_0t = 30\sqrt{20} \approx 134$ m
- Impact velocity: $v_x = 30$ m/s, $v_y = gt = 10\sqrt{20} \approx 44.7$ m/s
- $|\mathbf{v}| = \sqrt{(30^2 + 44.7^2)} \approx 54$ m/s

ADVANCED TOPICS FOR JEE

1. Variable Acceleration

- **Non-uniform motion** in 2D
- **Integration approach** for complex motion
- **Piece-wise analysis** for changing conditions

2. Relative Motion Applications

- **Rain and wind problems**
- **River crossing scenarios**
- **Motion in accelerated reference frames**

3. Advanced Projectile Motion

- **Projectile on inclined plane**
- **Motion with air resistance**
- **Trajectory optimization problems**

4. Complex Circular Motion

- **Vertical circular motion**
- **Banking of curves**

- **Combination of motions**

ERROR ANALYSIS IN 2D MOTION

Common Mistakes

1. Vector Addition Errors

- **Adding magnitudes instead of vectors**
- **Wrong component calculation**
- **Forgetting direction in final answer**

2. Projectile Motion Errors

- **Using wrong sign for gravity**
- **Confusing horizontal and vertical components**
- **Applying vertical motion equations to horizontal motion**

3. Circular Motion Confusion

- **Thinking uniform circular motion has no acceleration**
- **Confusing centripetal with tangential acceleration**
- **Wrong direction for centripetal acceleration**

Prevention Strategies

1. **Always draw vector diagrams**
2. **Set up coordinate system clearly**
3. **Work with components systematically**
4. **Check physical reasonableness of answers**

EXPERIMENTAL CONNECTIONS

1. Projectile Motion Experiments

- **Ball trajectory tracking**
- **Optimal angle determination**
- **Effect of initial conditions**

2. Circular Motion Studies

- **Centripetal force measurement**
- **Period and frequency relationships**
- **Banking angle optimization**

3. Vector Addition Verification

- **Force table experiments**
- **Resultant vector measurement**
- **Graphical vs analytical comparison**

EXAM SPRINT - FINAL CHECKLIST

Master These Concepts:

- ✓ Vector addition and subtraction
- ✓ Component resolution and magnitude calculation
- ✓ Projectile motion formulas and applications
- ✓ Circular motion relationships
- ✓ Motion analysis in 2D coordinate systems

Key Formulas to Memorize:

- ✓ $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, $|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2)}$
- ✓ $R = (v_0^2 \sin 2\theta)/g$, $h_m = (v_0^2 \sin^2 \theta)/(2g)$
- ✓ $a^c = v^2/R = \omega^2 R$, $v = \omega R$

Problem-Solving Steps:

- ✓ Set coordinate system
- ✓ Resolve into components
- ✓ Apply equations independently to x,y
- ✓ Combine results vectorially
- ✓ Verify physical reasonableness

EXAM SPRINT - Master Motion in a Plane through systematic study of vector concepts, projectile motion analysis, and circular motion principles. Focus on component-wise analysis and vector addition techniques for JEE/NEET success.

Source: NCERT Physics Class 11, Chapter 3 - Comprehensive coverage for competitive exam preparation