Chapter 8: Mechanical Properties of Solids - Detailed Notes

8.1 INTRODUCTION

Real vs Ideal Bodies:

- **Rigid Body (Idealized)**: Hard solid object with definite shape and size that doesn't deform
- **Real Bodies**: Can be stretched, compressed, and bent under sufficient external forces
- Even steel bars deform when large forces are applied

Key Material Properties:

1. Elasticity

- **Definition**: Property by which a body tends to regain its original size and shape when applied force is removed
- **Elastic Deformation**: Temporary deformation that is recoverable
- **Example**: Helical spring returns to original length when released

2. Plasticity

- **Definition**: Property where body gets permanently deformed and doesn't return to original shape
- **Plastic Deformation**: Permanent, non-recoverable deformation
- **Examples**: Putty, mud (close to ideal plastics)

Engineering Applications:

Understanding elastic behavior is crucial for:

- Building design (steel, concrete properties)
- Bridge construction
- Automobile manufacturing
- Aerospace engineering (lightweight but strong materials)
- Artificial limb design
- Railway track design (I-beam shape)

Material Selection Questions:

- Why is glass brittle while brass is not?
- Why do railway tracks have I-shaped cross-sections?
- How to design lightweight but sufficiently strong structures?

8.2 STRESS AND STRAIN

Fundamental Concepts:

Stress

Definition: Restoring force developed per unit area when a body is deformed

```
Stress (σ) = Force (F) / Area (A)
σ = F/A
```

Units: N/m^2 or Pascal (Pa) **Dimensions**: $[ML^{-1}T^{-2}]$

Strain

Definition: Fractional change in dimension of a body due to applied force

Characteristics:

- Strain is dimensionless (ratio of similar quantities)
- No units
- Measures relative deformation

Types of Stress and Strain:

1. Longitudinal Stress and Strain

Tensile Stress:

- Applied forces stretch the body (pull apart)
- Forces perpendicular to cross-sectional area
- Body elongates

Compressive Stress:

- Applied forces compress the body (push together)
- Body shortens

Longitudinal Strain:

```
Longitudinal strain (ε) = Change in length (ΔL) / Original length (L)
ε = ΔL/L
```

2. Shearing Stress and Strain

Shearing Stress:

- Forces applied parallel to cross-sectional area
- Causes relative displacement between opposite faces
- No change in length, only shape changes

Shearing Strain:

```
Shearing strain = Relative displacement (Δx) / Length (L)
γ = Δx/L = tan θ ≈ θ (for small angles)
```

Where θ = angular displacement from vertical

Practical Example:

- Book pushed horizontally while pressed from top
- One face moves relative to opposite face

3. Hydraulic Stress and Strain

Hydraulic Stress:

- Uniform pressure applied perpendicular to all surfaces
- Body submerged in fluid under high pressure
- Equal to applied hydraulic pressure

Volume Strain:

```
Volume strain = Change in volume (ΔV) / Original volume (V)
= ΔV/V
```

Key Points:

- No change in shape, only volume changes
- Occurs in solids, liquids, and gases
- Internal restoring forces balance applied pressure

8.3 HOOKE'S LAW

Statement:

For small deformations, stress is directly proportional to strain

Mathematical Expression:

```
Stress ∝ Strain
Stress = k × Strain
```

Where k = modulus of elasticity (proportionality constant)

Characteristics:

- **Empirical Law**: Based on experimental observations
- **Valid for most materials**: Within elastic limit
- **Linear Relationship**: Straight-line relationship between stress and strain
- **Some exceptions**: Materials like rubber don't follow Hooke's law over entire range

Physical Significance:

- Provides foundation for quantitative analysis of material behavior
- Enables calculation of material properties

- Basis for engineering design calculations

8.4 STRESS-STRAIN CURVE

Experimental Setup:

- Test specimen (wire or cylinder) subjected to gradually increasing force
- Strain (fractional change in length) measured
- Applied force per unit area (stress) calculated
- Graph plotted: Stress vs Strain

Typical Stress-Strain Curve Analysis:

Region OA: Linear Elastic Region

- **Straight line relationship**: Hooke's law valid
- **Elastic behavior**: Body returns to original dimensions when load removed
- **Slope**: Gives Young's modulus
- **Reversible deformation**: No permanent change

Region AB: Non-linear Elastic Region

- **Curved relationship**: Stress and strain not proportional
- **Still elastic**: Body returns to original shape when unloaded
- **Point B**: Yield point or elastic limit

Point B: Yield Point (Elastic Limit)

- **Yield Strength (σ_y)**: Maximum stress for elastic behavior
- **Critical point**: Beyond this, permanent deformation begins

- **Material-specific**: Different for each material

Region BC: Plastic Deformation Region

- **Permanent set**: Body doesn't return to original dimensions
- **Rapid strain increase**: Large deformation with small stress increase
- **Irreversible**: Permanent structural changes in material

Point D: Ultimate Tensile Strength

- **Ultimate Strength (σu)**: Maximum stress material can withstand
- **Peak stress**: Highest point on stress-strain curve
- **Material limit**: Beyond this, failure begins

Region DE: Failure Region

- **Necking**: Local reduction in cross-sectional area
- **Fracture**: Material breaks at point E
- **Reduced stress**: Due to decreasing cross-sectional area

Material Classification:

Brittle Materials:

- **Characteristic**: Points D and E are close
- **Behavior**: Sudden failure with little plastic deformation
- **Examples**: Glass, cast iron, ceramics
- **Failure mode**: Sudden fracture

Ductile Materials:

- **Characteristic**: Points D and E are far apart
- **Behavior**: Significant plastic deformation before fracture
- **Examples**: Mild steel, copper, aluminum
- **Failure mode**: Gradual necking and failure

Special Materials:

Elastomers:

- **Examples**: Rubber, biological tissues (aorta)
- **Large elastic region**: Can be stretched to several times original length
- **Non-linear**: Don't obey Hooke's law over most of range
- **No clear plastic region**: Gradual transition to failure

8.5 ELASTIC MODULI

General Definition:

Elastic Modulus = Stress / Strain

The proportionality constant in Hooke's law, characteristic of material properties.

8.5.1 Young's Modulus (Y)

Definition:

Ratio of longitudinal stress to longitudinal strain

```
Young's Modulus (Y) = Tensile or Compressive Stress / Longitudinal Strain
Y = σ/ε = (F/A)/(ΔL/L) = (F × L)/(A × ΔL)
```

Units: N/m^2 or Pa

Dimensions: $[ML^{-1}T^{-2}]$

Physical Significance:

- **Stiffness measure**: Higher Y means stiffer material
- **Resistance to deformation**: Large Y requires large force for small deformation
- **Material property**: Independent of shape and size

Values for Common Materials:

Material	Young's Modulus (10^{11} N/m^2)	Yield Strength (10^6 N/m^2)
Steel	2.0	300
Copper	1.1	200
Aluminum	0.69	95
Glass	0.65	50
Wood	0.13	50

Engineering Implications:

- **Steel**: Preferred for heavy-duty machines and structures
- **High Y materials**: Require large forces for small deformation
- **Design consideration**: Material selection based on stiffness requirements

8.5.2 Shear Modulus (G)

Definition:

Ratio of shearing stress to shearing strain

Shear Modulus (G) = Shearing Stress / Shearing Strain
$G = \tau/\gamma = (F/A)/(\Delta x/L) = (F \times L)/(A \times \Delta x)$
$G = (F/A)/\theta = F/(A \times \theta)$

Alternative Names:

- Modulus of rigidity
- Rigidity modulus

Units: N/m^2 or Pa

Relationship with Young's Modulus:

For most materials: **$G \approx Y/3$**

Values for Common Materials:

Material	Shear Modulus (10^9 N/m^2)
Steel	84
Tungsten	150
Copper	42
Aluminum	25
Glass	23

Applications:

- **Torsion analysis**: Shafts under twisting loads
- **Shear deformation**: Structures under lateral forces
- **Material selection**: For applications involving shear stress

8.5.3 Bulk Modulus (B)

Definition:

Ratio of hydraulic stress to volume strain

$$\text{Bulk Modulus (B)} = -\text{Hydraulic Stress} / \text{Volume Strain}$$
$$B = -p/(\Delta V/V)$$

Negative Sign Significance:

- Increase in pressure causes decrease in volume
- Ensures B is always positive for stable systems
- If $p > 0$, then $\Delta V < 0$

Units: N/m^2 or Pa

Compressibility (k):

$$\text{Compressibility (k)} = 1/B = -(1/\Delta p) \times (\Delta V/V)$$

Physical meaning: Fractional change in volume per unit pressure increase

Values for Different States:

Material	Bulk Modulus (10^9 N/m^2)
Solids	
Steel	160
Copper	140
Glass	37
Liquids	
Water	2.2
Mercury	25
Gases	
Air (STP)	1.0×10^{-4}

Key Observations:

- **Solids**: Least compressible (largest B)
- **Liquids**: Moderately compressible
- **Gases**: Most compressible (smallest B)
- **Ratio**: Gases ~10^6 times more compressible than solids

Physical Explanation:

- **Solids**: Tight atomic coupling resists volume change
- **Liquids**: Moderate intermolecular forces
- **Gases**: Weak intermolecular coupling, easily compressed

8.5.4 Poisson's Ratio (ν)

Definition:

Ratio of lateral strain to longitudinal strain

$$\text{Poisson's Ratio } (\nu) = \text{Lateral Strain} / \text{Longitudinal Strain}$$
$$\nu = (\Delta d/d) / (\Delta L/L) = (\Delta d/\Delta L) \times (L/d)$$

Characteristics:

- **Dimensionless**: Pure number, no units
- **Material property**: Depends only on material nature
- **Typical values**: 0.28-0.30 for steel, 0.33 for aluminum

Physical Meaning:

- When material is stretched longitudinally, it contracts laterally
- Quantifies the coupling between perpendicular deformations
- Related to atomic structure and bonding

8.5.5 Elastic Potential Energy

Work Done in Stretching:

When wire is stretched, work done against interatomic forces:

$$\text{Force } F = YA(l/L)$$
$$\text{Work done } dW = F \times dl = (YAl/L) \times dl$$

Total Work for elongation from 0 to l:

$$W = \int_0^l (YAl/L)\, dl = (YA/L) \times (l^2/2) = (YAl^2)/(2L)$$

Alternative Forms:

$$W = (1/2) \times Y \times \text{strain}^2 \times \text{Volume}$$
$$W = (1/2) \times \text{stress} \times \text{strain} \times \text{Volume}$$

Elastic Potential Energy per Unit Volume:

$$u = (1/2) \times \sigma \times \varepsilon = (1/2) \times \text{stress} \times \text{strain}$$

Physical Significance:

- Energy stored in deformed material
- Recoverable when deformation is removed
- Basis for spring energy calculations

8.6 APPLICATIONS OF ELASTIC BEHAVIOR

Structural Engineering Applications:

1. Crane Design

Problem: Design crane rope for 10-tonne capacity

Solution Approach:

- Avoid permanent deformation: stay within elastic limit
- Use yield strength for safety calculation
- Safety factor: ~10 times working load

Calculation:

Area required: $A \geq W/\sigma y = Mg/\sigma y$

$A \geq (10^4\ \text{kg} \times 9.8\ \text{m/s}^2)/(300 \times 10^6\ \text{N/m}^2)$

$A \geq 3.3 \times 10^{-4}\ \text{m}^2$

Radius ≥ 1 cm (with safety factor: 3 cm)

Practical Design:

- Multiple thin wires braided together
- Flexibility and strength combination
- Manufacturing ease

2. Beam Design

Loading Configuration: Beam supported at ends, loaded at center

Deflection Formula:

$$\delta = Wl^3/(4bd^3Y)$$

Where:

- W = load, l = length, b = breadth, d = depth
- Y = Young's modulus

Design Optimization:

- **Material selection**: Use high Y materials
- **Dimensional optimization**: Depth d more effective than breadth b
- **Cross-section shape**: I-beam provides optimal strength-to-weight ratio

I-Beam Advantages:

- Large load-bearing surface (flanges)
- Sufficient depth to prevent bending
- Reduced weight without sacrificing strength
- Cost-effective design
- Prevents buckling under lateral loads

3. Column Design

Load Distribution:

- Rounded ends: Less load capacity
- Distributed ends: Better load distribution
- Shape affects load-bearing capacity

Geological Applications:

Mountain Height Limitation

Physical Analysis:

- Material at mountain base under non-uniform compression
- Shearing stress component exists
- Maximum height limited by rock strength

Calculation:

Shearing stress ≈ hρg
Maximum stress = 30×10^7 N/m^2
h = $(30 \times 10^7)/(3 \times 10^3 \times 10)$ = 10 km

Result: Maximum mountain height ~10 km (consistent with Mt. Everest)

Safety Considerations:

Design Principles:

- **Safety Factor**: 10× working load typical
- **Material Properties**: Know yield strength, ultimate strength
- **Loading Conditions**: Consider dynamic loads, environmental factors
- **Failure Modes**: Design against different failure mechanisms

Material Selection Criteria:

- **Strength requirements**: Yield and ultimate strength
- **Stiffness requirements**: Young's modulus
- **Environmental resistance**: Corrosion, temperature effects
- **Cost considerations**: Economic feasibility
- **Manufacturing requirements**: Machinability, weldability

SUMMARY OF KEY FORMULAS

Stress and Strain:

- **Stress**: $\sigma = F/A$
- **Longitudinal Strain**: $\varepsilon = \Delta L/L$
- **Shearing Strain**: $\gamma = \Delta x/L = \tan\theta \approx \theta$
- **Volume Strain**: $\Delta V/V$

Hooke's Law:

- **General**: Stress = k × Strain
- **Linear region**: $\sigma \propto \varepsilon$

Elastic Moduli:

- **Young's Modulus**: $Y = \sigma/\varepsilon = (F \times L)/(A \times \Delta L)$
- **Shear Modulus**: $G = \tau/\gamma = (F \times L)/(A \times \Delta x)$
- **Bulk Modulus**: $B = -p/(\Delta V/V)$
- **Poisson's Ratio**: ν = (Lateral Strain)/(Longitudinal Strain)

Energy:

- **Elastic Potential Energy**: $U = (1/2) \times \sigma \times \varepsilon \times$ Volume
- **Energy Density**: $u = (1/2) \times \sigma \times \varepsilon$

Applications:

- **Beam Deflection**: $\delta = Wl^3/(4bd^3Y)$
- **Safety Design**: $A \geq W/\sigma y$
- **Mountain Height**: $h \leq \sigma max/(\rho g)$