

Chapter 9: Mechanical Properties of Fluids - Detailed Notes

9.1 INTRODUCTION

What Are Fluids?

Definition: Substances that can flow (liquids and gases) - distinguished from solids by their ability to change shape

Key Characteristics:

- **Flow Property:** Fundamental distinguishing feature from solids
- **No Definite Shape:** Take shape of container (unlike solids)
- **Volume Properties:**
 - Solids & liquids: Fixed volume under atmospheric pressure
 - Gases: Fill entire container volume
- **Low Shear Resistance:** Shape changes with very small shear stress (million times smaller than solids)

Everyday Importance:

- Earth's atmosphere envelope
- 2/3 of Earth's surface covered with water
- Mammalian bodies mostly water
- All biological processes mediated by fluids

Fluid vs Solid Properties

Property	Solids	Liquids	Gases
Shape	Definite	Container shape	Container shape
Volume	Fixed	Fixed	Fills container
Compressibility	Very low	Very low	High
Shear resistance	High	Very low	Very low

9.2 PRESSURE

Basic Concept

Physical Intuition:

- Sharp needle pierces skin, blunt spoon doesn't (same force)
- Elephant would crack ribs, but wooden plank distributes force
- **Key Principle:** Both force magnitude and contact area matter

Definition: Force per unit area acting normal to surface

Average Pressure: $P_{av} = F/A$

Exact Pressure: $P = \lim(\Delta A \rightarrow 0) \Delta F / \Delta A$

Properties:

- **Scalar Quantity:** No direction (only normal force component matters)
- **Dimensions:** $[ML^{-1}T^{-2}]$
- **SI Unit:** Pascal (Pa) = $N\ m^{-2}$
- **Common Units:**

- $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$
- $1 \text{ bar} = 10^5 \text{ Pa}$
- $1 \text{ torr} = 133 \text{ Pa}$

Fluid at Rest - Force Direction

Key Principle: Fluid at rest exerts only normal forces on surfaces

Proof: If force had parallel component $\rightarrow$ fluid would flow $\rightarrow$ contradicts "at rest" condition

Density

Definition: Mass per unit volume

$$\rho = m/V$$

Properties:

- **Dimensions:** $[ML^{-3}]$
- **SI Unit:** kg m^{-3}
- **Liquids:** Nearly constant (incompressible)
- **Gases:** Varies significantly with pressure
- **Reference:** Water at $4^\circ\text{C} = 1.0 \times 10^3 \text{ kg m}^{-3}$

9.2.1 Pascal's Law

Statement

Pressure Principle: Pressure in fluid at rest is same at all points at same height

Extended Form: External pressure applied to enclosed fluid is transmitted undiminished equally in all directions

Mathematical Proof

Consider triangular fluid element ABC-DEF at rest:

- Forces on three faces must balance
- Pressures P_a , P_b , P_c on areas A_a , A_b , A_c
- Force equilibrium: $F_b \sin\theta = F_c$, $F_b \cos\theta = F_a$
- Area geometry: $A_b \sin\theta = A_c$, $A_b \cos\theta = A_a$
- **Result:** $P_a = P_b = P_c$

Conclusion: Pressure is same in all directions at any point in fluid at rest

9.2.2 Variation of Pressure with Depth

Derivation

Consider cylindrical fluid element:

- Height: h , Base area: A
- Pressures: P_1 (top), P_2 (bottom)
- Weight: $mg = \rho Vg = \rho hAg$
- Vertical force balance: $(P_2 - P_1)A = \rho hAg$

Result:

$$P_2 - P_1 = \rho gh$$

Atmospheric Reference

For fluid open to atmosphere:

$$P = P_a + \rho gh$$

Where:

- P_a = atmospheric pressure
- h = depth below surface
- **Gauge Pressure:** $P_g = P - P_a = \rho gh$

Key Insights:

- Pressure depends only on vertical height difference
- Independent of container shape (hydrostatic paradox)
- Same pressure at all points at same horizontal level

9.2.3 Atmospheric Pressure and Gauge Pressure

Torricelli's Mercury Barometer

Principle: Atmospheric pressure balances mercury column

$$P_a = \rho_{\text{Hg}} \times g \times h$$

Standard Result: $h \approx 76 \text{ cm Hg}$ at sea level

Units:

- $1 \text{ torr} = 1 \text{ mm Hg} = 133 \text{ Pa}$
- $1 \text{ bar} = 10^5 \text{ Pa}$
- $1 \text{ millibar} = 100 \text{ Pa}$

Manometer

Open Tube Manometer: Measures gauge pressure

- U-tube with one end open to atmosphere
- Other end connected to system
- **Reading:** $P_{\text{gauge}} = \rho gh$ (proportional to height difference)

9.2.4 Hydraulic Machines

Pascal's Principle Application

Setup: Two pistons with areas A_1 , A_2 connected by fluid

Force Transmission:

$$P_1 = P_2$$

$$F_1/A_1 = F_2/A_2$$

$$F_2 = F_1 \times (A_2/A_1)$$

Mechanical Advantage: A_2/A_1

Applications:

- **Hydraulic Lift:** Small force lifts heavy loads
- **Hydraulic Brakes:** Foot pressure $\rightarrow$ wheel braking force
- **Advantage:** Equal pressure to all wheels

Volume Conservation: $A_1 L_1 = A_2 L_2$ (for incompressible fluid)

9.3 STREAMLINE FLOW

Flow Characteristics

Steady Flow: Velocity at each point constant in time

- Particle paths don't cross
- Flow pattern stationary in time
- Individual particles may accelerate along path

Streamline: Curve whose tangent gives fluid velocity direction at each point

Equation of Continuity

Mass Conservation Principle: Mass flow rate constant throughout pipe

Derivation:

- Mass crossing area A in time Δt : $\Delta m = \rho A v \Delta t$
- Same mass flows through all cross-sections
- **Result:** $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

For Incompressible Fluids:

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

$$A v = \text{constant (volume flow rate)}$$

Physical Meaning:

- Narrower sections $\rightarrow$ higher velocity
- Wider sections $\rightarrow$ lower velocity
- Streamlines closer together $\rightarrow$ higher speed

Flow Types

Laminar Flow: Smooth, parallel streamlines **Turbulent Flow:** Chaotic, irregular motion with eddies

Critical Speed: Transition point from laminar to turbulent

9.4 BERNOULLI'S PRINCIPLE

Statement

Along a streamline, sum of pressure, kinetic energy per unit volume, and potential energy per unit volume remains constant

Derivation

Work-Energy Theorem Applied: Consider fluid element moving from position 1 to 2:

- Work by pressure forces: $W = (P_1 - P_2)\Delta V$
- Change in kinetic energy: $\Delta K = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$
- Change in potential energy: $\Delta U = \rho g\Delta V(h_2 - h_1)$

Energy Conservation: $W = \Delta K + \Delta U$

Bernoulli's Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

General Form:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Assumptions and Limitations

Valid For:

- Non-viscous fluids (no internal friction)
- Incompressible fluids
- Steady (non-turbulent) flow

Limitations:

- Real fluids have viscosity → energy loss
- Compressible fluids store elastic energy
- Turbulent flow has fluctuating pressure/velocity

9.4.1 Speed of Efflux: Torricelli's Law

Tank with Hole

Setup: Tank with hole at depth h below surface **Assumptions:** Tank area $\gg$ hole area ($v_2 \approx 0$ at surface)

Applying Bernoulli: At hole: $P_1 = P_a$ At surface: $P_2 = P$ (tank pressure) Height difference: h

General Result:

$$v_1^2 = 2gh + 2(P - P_a)/\rho$$

Open Tank ($P = P_a$):

$$v = \sqrt{2gh}$$

Torricelli's Law: Same as free fall speed!

Applications:

- **Rocket Propulsion:** High container pressure
- **Water Flow:** Gravity-driven systems

9.4.2 Dynamic Lift

Magnus Effect (Spinning Ball)

Non-spinning Ball: Symmetric flow → no lift force **Spinning Ball:** Asymmetric flow

- Air dragged by spinning surface
- Higher velocity on one side
- Lower pressure on high-velocity side
- **Net Force:** Perpendicular to motion direction

Aerofoil (Aircraft Wing)

Design: Wing shape creates velocity difference

- **Upper Surface:** Higher velocity, lower pressure
- **Lower Surface:** Lower velocity, higher pressure
- **Net Result:** Upward lift force balances weight

Pressure Difference:

$$\Delta P = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\text{Lift Force} = \Delta P \times \text{Wing Area}$$

9.5 VISCOSITY

Physical Origin

Internal Friction: Resistance to relative motion between fluid layers **Mechanism:** Adjacent layers pull/push each other during flow

Viscous Flow Model:

- Top plate moves with velocity v
- Bottom plate fixed
- Velocity varies linearly: $v(y) = vy/l$
- Each layer experiences forces from neighbors

Coefficient of Viscosity

Definition: Ratio of shear stress to strain rate

$$\eta = (\text{Shear Stress})/(\text{Strain Rate})$$

$$\eta = (F/A)/(v/l)$$

Units:

- **SI:** poiseuille (Pl) = $\text{N s m}^{-2} = \text{Pa s}$
- **Dimensions:** $[\text{ML}^{-1}\text{T}^{-1}]$

Temperature Dependence:

- **Liquids:** η decreases with temperature (more mobile molecules)
- **Gases:** η increases with temperature (more random motion)

Typical Values

Fluid	Temperature (°C)	Viscosity (mPl)
Water	20	1.0
Water	100	0.3
Blood	37	2.7
Machine Oil	16	113
Glycerine	20	830
Air	0	0.017

9.5.1 Stokes' Law

Statement

Viscous drag force on sphere moving through fluid:

$$F = 6\pi\eta av$$

Where:

- η = viscosity of fluid
- a = radius of sphere
- v = velocity of sphere

Terminal Velocity

Force Balance: Weight = Buoyant force + Viscous drag

$$(4\pi/3)a^3(\rho - \sigma)g = 6\pi\eta av_t$$

Terminal Velocity:

$$v_t = \frac{2a^2(\rho - \sigma)g}{9\eta}$$

Dependencies:

- **Proportional to:** a^2 (radius squared)
- **Inversely proportional to:** η (viscosity)
- Applications: Raindrop falling, sedimentation

9.6 SURFACE TENSION

Physical Origin

Molecular Perspective:

- **Interior Molecules:** Surrounded by neighbors, lower potential energy
- **Surface Molecules:** Fewer neighbors, higher potential energy
- **Energy Cost:** Creating surface area requires work
- **Minimum Area:** Liquids tend toward minimum surface area

Surface Energy and Surface Tension

Surface Energy: Extra energy per unit area of molecules at surface **Surface Tension:** Force per unit length acting in surface plane

Relationship: $S = \text{Surface Energy per unit area} = \text{Force per unit length}$

Measurement: For thin film with movable boundary of length l :

Work done = $F \times d = S \times (\text{area increase})$
 $S = F/(2l)$ [Factor 2 for two surfaces of film]

Properties:

- **Scalar:** No preferred direction in surface plane
- **Temperature Dependent:** Generally decreases with temperature
- **Interface Property:** Depends on materials on both sides

Typical Values

Liquid	Temp (°C)	Surface Tension (N/m)
Water	20	0.0727
Mercury	20	0.4355
Ethanol	20	0.0227
Oxygen	-183	0.0132

9.6.3 Angle of Contact

Definition: Angle between liquid surface tangent and solid surface (measured inside liquid)

Force Balance: Three interfacial tensions at contact line

$$S_{la} \cos \theta + S_{sl} = S_{sa}$$

Where: la = liquid-air, sl = solid-liquid, sa = solid-air

Contact Angle Types:

- **Acute Angle** ($\theta < 90^\circ$): Liquid wets solid (water on clean glass)

- **Obtuse Angle** ($\theta > 90^\circ$): Liquid doesn't wet solid (mercury on glass)

Applications:

- **Detergents**: Reduce contact angle (better wetting)
- **Waterproofing**: Increase contact angle (poor wetting)

9.6.4 Drops and Bubbles

Spherical Shape: Minimum surface area for given volume minimizes surface energy

Pressure Inside Spherical Drop: Energy balance for radius change Δr :

- Surface energy increase: $8\pi r \Delta r S_{la}$
- Work by pressure difference: $(P_i - P_o) 4\pi r^2 \Delta r$
- **Result**: $P_i - P_o = 2S_{la}/r$

Pressure Inside Bubble: Bubble has two surfaces (inner and outer):

$$P_i - P_o = 4S_{la}/r$$

9.6.5 Capillary Rise

Physical Mechanism: Curved meniscus creates pressure difference

Setup: Vertical tube of radius a in liquid **Meniscus Curvature**: Radius = $a/\cos \theta$ **Pressure**

Difference: $(P_i - P_o) = (2S \cos \theta)/a$

Force Balance:

$$\rho gh = (2S \cos \theta)/a$$

Capillary Rise Formula:

$$h = (2S \cos \theta) / (\rho g a)$$

Key Dependencies:

- **Inversely proportional to:** tube radius a
- **Depends on:** contact angle θ
- **Positive h :** Acute contact angle (liquid rises)
- **Negative h :** Obtuse contact angle (liquid depresses)

SUMMARY OF KEY FORMULAS

Pressure and Fluid Statics

- **Pressure:** $P = F/A$
- **Pressure with depth:** $P = P_a + \rho g h$
- **Pascal's Law:** Applied pressure transmitted equally
- **Hydraulic machines:** $F_2/F_1 = A_2/A_1$

Fluid Dynamics

- **Continuity equation:** $Av = \text{constant}$
- **Bernoulli's equation:** $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$
- **Torricelli's law:** $v = \sqrt{2gh}$

Wave Speeds

- **Transverse (string):** $v = \sqrt{T/\mu}$
- **Longitudinal (fluid):** $v = \sqrt{B/\rho}$

- **Sound in gas:** $v = \sqrt{\gamma P / \rho}$

Viscosity

- **Viscosity definition:** $\eta = (F/A)/(dv/dy)$
- **Stokes' law:** $F = 6\pi\eta av$
- **Terminal velocity:** $v_t = 2a^2(\rho - \sigma)g/(9\eta)$

Surface Tension

- **Surface tension:** $S = F/(2l)$
- **Contact angle:** $S_{la} \cos \theta + S_{sl} = S_{sa}$
- **Drop pressure:** $P_i - P_o = 2S/r$
- **Bubble pressure:** $P_i - P_o = 4S/r$
- **Capillary rise:** $h = (2S \cos \theta)/(\rho ga)$