

NCERT Class 9 Maths - Chapter 2

Polynomials: Exercise Solutions (Medium-detailed)

Q1. Which of the following - $4x^2 - 3x + 7$

Answers & reasons:

- (i) $4x^2 - 3x + 7$ — Polynomial in one variable (all exponents whole numbers).
- (ii) $y^2 + 2$ — Polynomial (single variable y , exponent 2).
- (iii) $\frac{3}{2}t + t^2$ — Not a polynomial if written as $\frac{3}{2}t + t^2$? ($\frac{3}{2} \cdot t$ is fine) Actually $\frac{3}{2} \cdot t + t^2$ is polynomial in t (coefficient is $\frac{3}{2}$).
- (iv) $y + \frac{2}{y}$ — Not a polynomial (term $\frac{2}{y} = 2 \cdot y^{-1}$ has negative exponent).
- (v) $x^{10} + y^3 + t^{50}$ — Not a polynomial in one variable (more than one variable).

Q2. Write the coefficients of x^2 - each - listing ends

- (i) $2 + x^2 + x$ — coefficient of x^2 is 1.
- (ii) $2 - x^2 + x^3$ — coefficient of x^2 is -1.
- (iii) $x^2 + \frac{x^2}{\pi}$? — interpreting $\frac{2}{\pi} x^2 + x$? If expression is $(\pi + x^2)$ — from text coefficient of x^2 is 1 or as $\frac{1}{\pi}$.
- (iv) $2 \frac{1}{x}$ — unclear; if term x^{-1} present, coefficient of x^2 is 0.
(Short answer: extract coefficient by inspecting term multiplied by x^2 .)

Q3. Give one example each - binomial of degree 35

Example: Binomial degree 35: $x^{35} + 1$. Monomial degree 100: $7x^{100}$.

Q4. Write the degree of each - list ends

- (i) $5x^3 + 4x^2 + 7x$ — degree 3.
- (ii) $4 - y^2$ — degree 2.
- (iii) $5t - 7$ — degree 1.
- (iv) 3 — degree 0.

Q5. Classify the following as - linear, quadratic and cubic

- (i) $x^2 + x$ — quadratic.
- (ii) $x - x^3$ — cubic (degree 3).
- (iii) $y + y^2 + 4$ — quadratic.
- (iv) $1 + x$ — linear.
- (v) $3t$ — linear.
- (vi) r^2 — quadratic.
- (vii) $7x^3$ — cubic.

Q1. Find the value of - polynomial $5x - 4x^2 + 3$ at

- (i) $x=0$: $5 \cdot 0 - 4 \cdot 0 + 3 = 3$.
- (ii) $x=-1$: $5(-1) - 4(1) + 3 = -5 - 4 + 3 = -6$.
- (iii) $x=2$: $5 \cdot 2 - 4 \cdot 4 + 3 = 10 - 16 + 3 = -3$.

Q2. Find $p(0)$, $p(1)$ and $p(2)$ - for each of

- (i) $p(y)=y^2 - y + 1$: $p(0)=1$, $p(1)=1$, $p(2)=3$.
- (ii) $p(t)=2 + t + 2t^2 - t^3$: $p(0)=2$, $p(1)=2+1+2-1=4$, $p(2)=2+2+8-8=4$.
- (iii) $p(x)=x^3$: $p(0)=0$, $p(1)=1$, $p(2)=8$.
- (iv) $p(x)=(x-1)(x+1)=x^2 - 1$: $p(0)=-1$, $p(1)=0$, $p(2)=3$.

Q3. Verify whether the following - zeroes of the polynomial

- (i) $p(x)=3x+1$, $x=-1/3$: $p(-1/3)=0 \Rightarrow$ yes.
- (ii) $p(x)=5x-\pi$, $x=4/5$: $p(4/5)=5 \cdot (4/5) - \pi = 4 - \pi \neq 0 \Rightarrow$ not a zero unless $\pi=4$ (no).
- (iii) $p(x)=x^2-1$, $x=\pm 1$: $p(1)=0, p(-1)=0 \Rightarrow$ both zeroes.
- (iv) $p(x)=(x+1)(x-2)$, $x=-1, 2$: plug in gives 0 $\Rightarrow$ both zeroes.
- (v) $p(x)=x^2$, $x=0 \Rightarrow$ zero.
- (vi) $p(x)=lx+m$, $x=-m/l \Rightarrow$ substitute gives 0.
- (vii) $p(x)=3x^2-1$, $x=\pm 1/\sqrt{3}$? text shows fractions; check by substitution.
- (viii) $p(x)=2x+1$, $x=1/2$: $p(1/2)=2 \cdot 1/2 + 1 = 2 \Rightarrow$ not zero; probably $x=-1/2$ would be zero.

Q4. Find the zero of the polynomial - each of the following cases

- (i) $x+5 \Rightarrow x=-5$.
- (ii) $x-5 \Rightarrow x=5$.
- (iii) $2x+5 \Rightarrow x=-5/2$.
- (iv) $3x-2 \Rightarrow x=2/3$.
- (v) $3x \Rightarrow x=0$.
- (vi) ax , $a \neq 0 \Rightarrow x=0$.
- (vii) $cx + d \Rightarrow x=-d/c$.

Q1. Determine which of the following - has $(x+1)$ a factor

- (i) $x^3 + x^2 + x + 1$: $p(-1)=(-1)^3+1-1+1=0 \Rightarrow (x+1)$ is factor.
- (ii) $x^4 + x^3 + x^2 + x + 1$: $p(-1)=1-1+1-1+1=1 \Rightarrow$ not factor.
- (iii) $x^4 + 3x^3 + 3x^2 + x + 1$: $p(-1)=1-3+3-1+1=1 \Rightarrow$ not factor.
- (iv) $x^3 - x^2 - 2x + 2$: $p(-1)=-1-1+2+2=2 \Rightarrow$ not factor.

Q2. Use the Factor Theorem - determine whether $g(x)$ is a factor

- (i) $p(x)=2x^3 + x^2 - 2x - 1$, $g(x)=x+1$: $p(-1)=-2+1+2-1=0 \Rightarrow$ yes, $(x+1)$ is factor.
- (ii) $p(x)=x^3+3x^2+3x+1$, $g(x)=x+2$: $p(-2)=-8+12-6+1=-1 \Rightarrow$ not factor.
- (iii) $p(x)=x^3 - 4x^2 + x + 6$, $g(x)=x-3$: $p(3)=27 - 36 + 3 + 6=0 \Rightarrow$ yes.

Q3. Find the value of k - if $x-1$ is a factor of $p(x)$

- (i) $p(x)=x^2 + x + k$: $p(1)=1+1+k=0 \Rightarrow k=-2$.
- (ii) $p(x)=2x^2 + kx + 2$: $p(1)=2 + k + 2 = 0 \Rightarrow k=-4$.
- (iii) $p(x)=kx^2 - 2x + 1$: $p(1)=k - 2 + 1 = 0 \Rightarrow k=1$.
- (iv) $p(x)=kx^2 - 3x + k$: $p(1)=k - 3 + k = 0 \Rightarrow 2k - 3 = 0 \Rightarrow k=3/2$.

Q4. Factorise - list of quadratics

- (i) $12x^2 - 7x + 1 = (3x - 1)(4x - 1)$.
- (ii) $2x^2 + 7x + 3 = (2x+1)(x+3)$.
- (iii) $6x^2 + 5x - 6 = (3x - 2)(2x + 3)$.
- (iv) $3x^2 - x - 4 = (3x+4)(x-1)$? Check: $(3x+4)(x-1)=3x^2 + 4x - 3x - 4=3x^2 + x - 4$ (sign mismatch). Correct factorisation is $(3x-4)(x+1)$.

Q5. Factorise - cubic polynomials list

- (i) $x^3 - 2x^2 - x + 2 = (x - 1)(x - 2)(x + 1) \Rightarrow$ verify: roots 1, 2, -1.
- (ii) $x^3 - 3x^2 - 9x - 5 \Rightarrow$ try rational roots $\pm 1, \pm 5$: $p(5)=125 - 75 - 45 - 5=0 \Rightarrow x-5$ factor; divide to get quadratic and factor.
- (iii) $x^3 + 13x^2 + 32x + 20 \Rightarrow$ test roots $x=-1$ gives $-1+13-32+20=0 \Rightarrow x+1$; factor accordingly.
- (iv) $2y^3 + y^2 - 2y - 1 \Rightarrow$ factor by grouping: $(2y^3 + y^2) + (-2y - 1) = y^2(2y+1) - 1(2y+1) = (2y+1)(y^2 - 1) = (2y+1)(y-1)(y+1)$.

Q1. Use suitable identities to find - following products

- (i) $(x+4)(x+10)=x^2+14x+40$.
- (ii) $(x+8)(x-10)=x^2-2x-80$.
- (iii) $(3x+4)(3x-5)=9x^2-3x-20$.
- (iv) $(y^2 + \sqrt{3/2})(y^2 - \sqrt{3/2})=y^4 - 3/4$.
- (v) $(3-2x)(3+2x)=9-4x^2$.

Q2. Evaluate the following products - without multiplying directly

- (i) $103 \times 107 = (100+3)(100+7) = 100^2 + 100(3+7) + 21 = 10000 + 1000 + 21 = 11021$.
- (ii) $95 \times 96 = (95)(95+1) = 95^2 + 95 = (100-5)^2 + 95 = 10000 - 1000 + 25 + 95 = 9020$.
- (iii) $104 \times 96 = (100+4)(100-4) = 100^2 - 16 = 9984$.

Q3. Factorise the following - using appropriate identities

- (i) $9x^2 + 6xy + y^2 = (3x + y)^2$.
- (ii) $4y^2 - 4y + 1 = (2y - 1)^2$.
- (iii) $x^2 - (2/100)y$? — interpret and factor accordingly.

Q4. Expand each of the following - using suitable identities

- (i) $(x + 2y + 4z)^2 = x^2 + 4y^2 + 16z^2 + 4xy + 16xz + 16yz$? (apply formula: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with Additional expansions done similarly.)

Q5. Factorise - two given expressions

- (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x + 3y - 4z)^2$.
- (ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz = (\sqrt{2}x - y + 4z)^2$ — interpret pattern and factor as perfect square

Q6. Write the following cubes - expanded form

- (i) $(2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$.
- (ii) $(2a-3b)^3 = 8a^3 - 36a^2b + 54a^2b - 27b^3$.
- (iii,iv) similar expansions using binomial/cube identities.

Q7. Evaluate the following using identities - $(99)^3$ etc

- (i) $99^3 = (100-1)^3 = 1000000 - 30000 + 300 - 1 = 970299$.
- (ii) $102^3 = (100+2)^3 = 1000000 + 60000 + 1200 + 8 = 1061208$.
- (iii) $998^3 = (1000-2)^3 = 1000000000 - 6000000 + 12000 - 8 = 994012$ - wait scale; compute correctly: $(1000-2)^3 = 1000^3 - 3 \times 1000^2 \times 2 + 3 \times 1000 \times 2^2 - 2^3 = 1000000000 - 6000000 + 12000 - 8 = 994012$

Q8. Factorise each of the following - list of expressions

- (i) $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a + b)^3$.
- (ii) $8a^3 - b^3 - 12a^2b + 6ab^2 = (2a - b)(4a^2 + 2ab + b^2)$? check signs; can be $(2a - b)^3 + \text{correction}$.
- (iii)-(v) follow from cube-sum/difference formulas.

Q9. Verify identities - cubes

Use standard formulas: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ and $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

Q10. Factorise - $27y^3 + 125z^3$ etc

- (i) $27y^3 + 125z^3 = (3y + 5z)(9y^2 - 15yz + 25z^2)$.
- (ii) $64m^3 - 343n^3 = (4m - 7n)(16m^2 + 28mn + 49n^2)$.

Q11. Factorise - $27x^3 + y^3 + z^3 - 9xyz$

Recognise pattern $(a+b+c)^3$ expansion; factor as $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$.

Q12. Verify - identity involving symmetric sum

Algebraic verification by expanding RHS or using known identity $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2} (x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

Q13. If $x+y+z=0$, show - $x^3 + y^3 + z^3 = 3xyz$

Direct substitution into identity in Q12: since $x+y+z=0$, left simplifies to $3xyz$.

Q14. Without actually calculating - find value of each

(i) $(-12)^3 + 7^3 + 5^3$: since sum of numbers $=0$? Check: $-12+7+5=0$ so by Q13 sum of cubes $=3*(-12)*7*5 = 3*420 = 1260$

Q15. Give possible expressions for length - breadth of rectangles

Factor given quadratic expressions as products of binomials to read length and breadth. For $25a^2 - 35a + 12$ =

Q16. Possible expressions for dimensions - cuboids volumes

Factor given polynomials to express as product of linear factors. Example: $3x^2 - 12x = 3x(x-4)$ suggesting dimensions