Motion in a Plane - NCERT Exercise Solutions

Chapter 3 - Class 11 Physics

NCERT TEXTBOOK EXERCISES

3.1 State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Answer:

- **Volume** - Scalar (has only magnitude)
- **Mass** - Scalar (has only magnitude)
- **Speed** - Scalar (magnitude of velocity)
- **Acceleration** - Vector (has magnitude and direction)
- **Density** - Scalar (has only magnitude)
- **Number of moles** - Scalar (has only magnitude)
- **Velocity** - Vector (has magnitude and direction)
- **Angular frequency** - Scalar (has only magnitude)
- **Displacement** - Vector (has magnitude and direction)
- **Angular velocity** - Vector (has magnitude and direction)

3.2 Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity,

magnetic moment, relative velocity.

Answer: The two scalar quantities are:

1. **Work** - Scalar (has only magnitude)
2. **Current** - Scalar (has only magnitude)

All others are vector quantities as they have both magnitude and direction.

3.3 Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Answer: The only vector quantity is: **Impulse** - Vector (has magnitude and direction, equal to change in momentum)

All others are scalar quantities having only magnitude.

3.4 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: (a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Answer: (a) Adding any two scalars - Not always meaningful

- Only meaningful if scalars have same dimensions/units
- Example: 5 m + 3 m = 8 m (meaningful)
- Example: 5 kg + 3 s (not meaningful)

(b) Adding a scalar to a vector of the same dimensions - **Not meaningful**

- Cannot add scalar to vector even with same dimensions
- Different mathematical entities that don't combine by simple addition

(c) Multiplying any vector by any scalar - **Always meaningful**

- Results in a vector with magnitude scaled by scalar
- Direction same (if scalar positive) or opposite (if scalar negative)

(d) Multiplying any two scalars - **Always meaningful**

- Follows ordinary multiplication rules
- Dimensions multiply: $[\text{scalar}_1] \times [\text{scalar}_2]$

(e) Adding any two vectors - **Not always meaningful**

- Only meaningful if vectors have same dimensions
- Must use vector addition rules (parallelogram/triangle law)

(f) Adding a component of a vector to the same vector - **Not meaningful**

- Component is scalar, original is vector
- Cannot add scalar to vector

3.5 Read each statement below carefully and state with reasons, if it is true or false: (a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle, (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either

greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

Answer: (a) The magnitude of a vector is always a scalar - **TRUE**

- Magnitude |**A**| has only numerical value (positive)
- No directional property

(b) Each component of a vector is always a scalar - **TRUE**

- Components (Ax, Ay, Az) are numbers (can be positive, negative, or zero)
- No directional property by themselves

(c) The total path length is always equal to the magnitude of the displacement vector - **FALSE**

- Path length ≥ |displacement|
- Equal only for straight-line motion without direction change
- In general, path length > |displacement|

(d) Average speed ≥ magnitude of average velocity - **TRUE**

- Average speed = total path length / time
- |Average velocity| = |displacement| / time
- Since path length ≥ |displacement|, the inequality holds

(e) Three vectors not lying in a plane can never add up to give a null vector - **FALSE**

- Three non-coplanar vectors can add to give null vector
- Example: Three vectors along x, y, z axes with appropriate magnitudes
- Condition: **A** + **B** + **C** = **0** is possible in 3D

3.6 Establish the following vector inequalities geometrically or otherwise: (a) |a + b| ≤ |a| + |b|, (b) |a + b| ≥ ||a| - |b||, (c) |a - b| ≤ |a| + |b|, (d) |a - b| ≥ ||a| - |b||. When does the equality sign above apply?

Answer:

(a) |a + b| ≤ |a| + |b| (Triangle Inequality)

- **Geometrically:** In triangle, sum of two sides ≥ third side
- **Equality when: a** and **b** are in same direction (θ = 0°)
- **Using cosine law:** $|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}||\mathbf{b}|\cos\theta$
- Maximum when cos θ = 1 (θ = 0°)

(b) |a + b| ≥ ||a| - |b||

- **Equality when: a** and **b** are in opposite directions (θ = 180°)
- When θ = 180°, cos θ = -1, giving |**a** + **b**| = ||**a**| - |**b**||

(c) |a - b| ≤ |a| + |b|

- Since **a** - **b** = **a** + (-**b**)
- Applying triangle inequality: |**a** + (-**b**)| ≤ |**a**| + |-**b**| = |**a**| + |**b**|
- **Equality when: a** and **b** are in opposite directions

(d) |a - b| ≥ ||a| - |b||

- **Equality when: a** and **b** are in same direction

3.7 Given a + b + c + d = 0, which of the following statements are correct: (a) a, b, c, and d must each be a null vector, (b) The magnitude of (a + c) equals the magnitude of (b + d), (c) The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d, (d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear?

Answer: Given: **a** + **b** + **c** + **d** = **0** This means: **a** + **c** = -(**b** + **d**)

(a) FALSE - Individual vectors need not be null vectors

- Only their vector sum is zero

(b) TRUE - Since **a** + **c** = -(**b** + **d**)

- |**a** + **c**| = |-(**b** + **d**)| = |**b** + **d**|

(c) TRUE - From **a** = -(**b** + **c** + **d**)

- |**a**| = |**b** + **c** + **d**| ≤ |**b**| + |**c**| + |**d**| (triangle inequality)

(d) TRUE - Since **b** + **c** = -(**a** + **d**)

- **b** + **c** lies along -(**a** + **d**), which is in the plane of **a** and **d**

3.8 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge and reach a point Q diametrically opposite to P following different paths. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skated?

Answer: Given: Circular ground radius = 200 m, P and Q are diametrically opposite

Displacement vector: Since P and Q are diametrically opposite

- Displacement = straight line distance PQ = diameter = 2 × 200 = 400 m
- **Same for all three girls** = 400 m

Path lengths:

- Girl 1 (straight line): 400 m
- Girl 2 (semicircular arc): π × 200 = 628.3 m
- Girl 3 (longer curved path): > 628.3 m

Answer: Displacement magnitude = 400 m for each girl Path length equals displacement only for **Girl 1** (straight line path).

3.9 A cyclist starts from centre O of circular park of radius 1 km, reaches edge P, then cycles along circumference, and returns to centre along QO. If the round trip takes 10 min, find: (a) net displacement, (b) average velocity, (c) average speed.

Answer: Given: Radius = 1 km, Time = 10 min = 600 s

(a) Net displacement:

- Initial position: Centre O
- Final position: Centre O
- **Net displacement = 0**

(b) Average velocity:

- Average velocity = Net displacement / Total time = 0/600 = **0**

(c) Average speed: Path calculation:

- O to P: 1 km (radius)

- P to Q along circumference: (1/4) × 2π × 1 = π/2 km
- Q to O: 1 km (radius)
- Total path = 1 + π/2 + 1 = 2 + π/2 ≈ 3.57 km

Average speed = Total path / Total time = 3.57 km / 10 min = 0.357 km/min = 5.95 m/s

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3.10 On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Answer: Analysis: The motorist follows a regular hexagonal path (360°/60° = 6 sides) Each side = 500 m

At third turn:

- Path covered: 3 × 500 = 1500 m
- Position: Forms equilateral triangle with starting point
- Displacement = 500√3 ≈ 866 m
- Ratio = 866/1500 = 0.577

At sixth turn:

- Path covered: 6 × 500 = 3000 m
- Position: Back to starting point (completed hexagon)
- Displacement = 0 m
- Ratio = 0/3000 = 0

At eighth turn:

- Path covered: 8 × 500 = 4000 m
- Position: Same as second turn
- Displacement = 500 m
- Ratio = 500/4000 = 0.125

3.11 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Answer: Given:

- Displacement = 10 km (straight line distance)
- Path length = 23 km (actual distance traveled)
- Time = 28 min = 28/60 h = 7/15 h

(a) Average speed: Average speed = Total path length / Time = 23 km / (7/15) h = 23 × 15/7 = **49.3 km/h**

(b) Magnitude of average velocity: |Average velocity| = |Displacement| / Time = 10 km / (7/15) h = 10 × 15/7 = **21.4 km/h**

Are they equal? NO

- Average speed (49.3 km/h) > |Average velocity| (21.4 km/h)
- This is expected since path length > displacement magnitude

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3.12 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Answer: Given: Maximum height allowed = 25 m, Initial speed = 40 m/s

Condition: Maximum height of trajectory ≤ 25 m

Maximum height formula: $h_m = (v_0^2 \sin^2 \theta)/(2g)$

Setting up inequality: $(40^2 \sin^2 \theta)/(2 \times 10) \leq 25$ $1600 \sin^2 \theta/20 \leq 25$ $80 \sin^2 \theta \leq 25$ $\sin^2 \theta \leq 25/80 = 5/16$ $\sin \theta \leq \sqrt{(5/16)} = \sqrt{5}/4$

For maximum range with this constraint: $\sin \theta = \sqrt{5}/4$, so $\cos \theta = \sqrt{(1 - 5/16)} = \sqrt{11}/4$

Maximum range: $R = (v_0^2 \sin 2\theta)/g = (v_0^2 \times 2 \sin \theta \cos \theta)/g$ $R = (1600 \times 2 \times \sqrt{5}/4 \times \sqrt{11}/4)/10 = (1600 \times 2 \times \sqrt{55}/16)/10$ $R = (1600 \times \sqrt{55}/8)/10 =$ **$60\sqrt{55} \approx 44.7$ m**

3.13 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Answer: Given: Maximum horizontal range = 100 m

For maximum range: $\theta = 45°$, so $R_{max} = v_0^2/g$ $100 = v_0^2/10$ $v_0^2 = 1000\ m^2/s^2$

For maximum height: Ball thrown vertically upward ($\theta = 90°$) $h_{max} = v_0^2/(2g) = 1000/(2 \times 10) =$ **50 m**

Relationship: Maximum height = (Maximum range)/2

3.14 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Answer: Given:

- Radius r = 80 cm = 0.8 m
- Number of revolutions = 14 in 25 s

Step 1: Find angular speed $\omega = (14 \times 2\pi)/25 = 28\pi/25$ rad/s

Step 2: Find centripetal acceleration $a_c = \omega^2 r = (28\pi/25)^2 \times 0.8$ $a_c = (784\pi^2/625) \times 0.8 = 627.2\pi^2/625 \approx$ **9.9 m/s²**

Direction: Always directed towards the center of the circular path (centripetal direction)

3.15 An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Answer: Given:

- Radius r = 1.00 km = 1000 m
- Speed v = 900 km/h = 900 × (5/18) = 250 m/s

Centripetal acceleration: $a_c = v^2/r = (250)^2/1000 = 62{,}500/1000 =$ **62.5 m/s²**

Comparison with gravity: $a_c/g = 62.5/9.8 =$ **6.38**

The centripetal acceleration is **6.38 times** the acceleration due to gravity.

3.16 Read each statement carefully and state with reasons if it is true or false: (a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre (b) The velocity vector of a particle at a point is always along the tangent to the path at that point (c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

Answer: (a) FALSE for general circular motion, TRUE for uniform circular motion

- In uniform circular motion: only centripetal acceleration (toward center)
- In non-uniform circular motion: both centripetal (toward center) and tangential components
- Net acceleration is along radius only for uniform circular motion

(b) TRUE

- Velocity is defined as dr/dt, which is always tangent to the path
- This is true for any type of motion, not just circular

(c) TRUE

- In uniform circular motion, acceleration magnitude is constant but direction keeps changing
- Over one complete cycle, the acceleration vector traces a circle
- Vector average over complete cycle = 0 (null vector)

3.17 The position of a particle is given by $\mathbf{r} = 3.0t\hat{\mathbf{i}} - 2.0t^2\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}}$ m where t is in seconds and the coefficients have proper units for r to be in metres. (a) Find $\mathbf{v}$ and $\mathbf{a}$ of the particle (b) What is the magnitude and direction of velocity at t = 2.0 s?

Answer: Given: $\mathbf{r} = 3.0t\hat{\mathbf{i}} - 2.0t^2\hat{\mathbf{j}} + 4.0\hat{\mathbf{k}}$

(a) Finding velocity and acceleration:

Velocity: $\mathbf{v} = d\mathbf{r}/dt = 3.0\hat{\mathbf{i}} - 4.0t\hat{\mathbf{j}} + 0\hat{\mathbf{k}} = \mathbf{3.0\hat{i} - 4.0t\hat{j}}$ m/s

Acceleration: $\mathbf{a} = d\mathbf{v}/dt = 0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}} + 0\hat{\mathbf{k}} = \mathbf{-4.0\hat{j}}$ m/s^2

(b) At t = 2.0 s: $\mathbf{v}(2.0) = 3.0\hat{\mathbf{i}} - 4.0(2.0)\hat{\mathbf{j}} = \mathbf{3.0\hat{i} - 8.0\hat{j}}$ m/s

Magnitude: $|\mathbf{v}| = \sqrt{(3.0^2 + (-8.0)^2)} = \sqrt{(9 + 64)} = \mathbf{\sqrt{73} \approx 8.54}$ **m/s**

Direction: $\tan \theta = v_y/v_x = -8.0/3.0 = -2.67$ $\theta = \tan^{-1}(-2.67) = \mathbf{-69.4°}$ (below positive x-axis)

3.18 A particle starts from origin at t = 0 s with velocity of 10.0ĵ m/s and moves in x-y plane with constant acceleration of (8.0î + 2.0ĵ) m/s². (a) At what time is the x-coordinate 16 m? What is the y-coordinate then? (b) What is the speed at that time?

Answer: Given:

- Initial position: $\mathbf{r_0} = 0$
- Initial velocity: $\mathbf{v_0} = 10.0\hat{\mathbf{j}}$ m/s
- Acceleration: $\mathbf{a} = 8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}$ m/s^2

Position equation: $\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}t + ½\mathbf{a}t^2$ $\mathbf{r} = 0 + 10.0t\hat{\mathbf{j}} + ½(8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})t^2$ $\mathbf{r} = 4.0t^2\hat{\mathbf{i}} + (10.0t + 1.0t^2)\hat{\mathbf{j}}$

(a) When x = 16 m: $x = 4.0t^2 = 16$ $t^2 = 4$, so **$t = 2.0$ s**

y-coordinate at t = 2.0 s: $y = 10.0(2.0) + 1.0(2.0)^2 = 20.0 + 4.0 =$ **24.0 m**

(b) Speed at t = 2.0 s: Velocity equation: $\mathbf{v} = \mathbf{v_0} + \mathbf{a}t = 10.0\hat{\mathbf{j}} + (8.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})t$ At t = 2.0 s: $\mathbf{v} = 10.0\hat{\mathbf{j}} + 16.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}} = \mathbf{16.0\hat{i} + 14.0\hat{j}}$ m/s

Speed: $|\mathbf{v}| = \sqrt{(16.0^2 + 14.0^2)} = \sqrt{(256 + 196)} = \mathbf{\sqrt{452} \approx 21.3}$ **m/s**

3.19 $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? What are the components of vector $\mathbf{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

Answer: For vector $\hat{i} + \hat{j}$:

- Magnitude: $|\hat{\mathbf{i}} + \hat{\mathbf{j}}| = \sqrt{(1^2 + 1^2)} = \mathbf{\sqrt{2}}$
- Direction: $\tan\theta = 1/1 = 1$, so $\theta =$ **45°** with x-axis

For vector $\hat{i} - \hat{j}$:

- Magnitude: $|\hat{\mathbf{i}} - \hat{\mathbf{j}}| = \sqrt{(1^2 + (-1)^2)} = \mathbf{\sqrt{2}}$
- Direction: $\tan\theta = -1/1 = -1$, so $\theta =$ **-45°** with x-axis

Components of $\mathbf{A} = 2\hat{i} + 3\hat{j}$:

Unit vectors in required directions:

- $\hat{\mathbf{u}}_1 = (\hat{\mathbf{i}} + \hat{\mathbf{j}})/\sqrt{2}$
- $\hat{\mathbf{u}}_2 = (\hat{\mathbf{i}} - \hat{\mathbf{j}})/\sqrt{2}$

Component along $(\hat{i} + \hat{j})$: $A_1 = \mathbf{A} \cdot \hat{\mathbf{u}}_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot ((\hat{\mathbf{i}} + \hat{\mathbf{j}})/\sqrt{2}) = (2 + 3)/\sqrt{2} = \mathbf{5/\sqrt{2} = 5\sqrt{2}/2}$

Component along $(\hat{i} - \hat{j})$:
$A_2 = \mathbf{A} \cdot \hat{\mathbf{u}}_2 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot ((\hat{\mathbf{i}} - \hat{\mathbf{j}})/\sqrt{2}) = (2 - 3)/\sqrt{2} = \mathbf{-1/\sqrt{2} = -\sqrt{2}/2}$

3.20 For any arbitrary motion in space, which of the following relations are true: (a) $\mathbf{v}_{ave} = (1/2)(\mathbf{v}(t_1) + \mathbf{v}(t_2))$ (b) $\mathbf{v}_{ave} = [\mathbf{r}(t_2) - \mathbf{r}(t_1)]/(t_2 - t_1)$ (c) $\mathbf{v}(t) = \mathbf{v}(0) + \mathbf{a}t$ (d) $\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}(0)t + (1/2)\mathbf{a}t^2$ (e) $\mathbf{a}_{ave} = [\mathbf{v}(t_2) - \mathbf{v}(t_1)]/(t_2 - t_1)$

Answer: (a) FALSE - This is true only for constant acceleration

- For arbitrary motion, average velocity ≠ arithmetic mean of initial and final velocities

(b) TRUE - This is the definition of average velocity

- Always true regardless of type of motion

(c) FALSE - This is true only for constant acceleration

- For arbitrary motion, acceleration can vary with time

(d) FALSE - This is true only for constant acceleration

- For arbitrary motion, requires integration: $\mathbf{r}(t) = \mathbf{r}(0) + \int \mathbf{v}(t)dt$

(e) TRUE - This is the definition of average acceleration

- Always true regardless of type of motion

3.21 Read each statement carefully and state with reasons and examples if it is true or false: A scalar quantity is one that (a) is conserved in a process (b) can never take negative values (c) must be dimensionless (d) does not vary from one point to another in space (e) has the same value for observers with different orientations of axes

Answer: (a) FALSE - Scalar quantities can change in processes

- Example: Temperature changes in heating/cooling processes
- Kinetic energy changes when speed changes

(b) FALSE - Scalar quantities can be negative

- Examples: Temperature (-10°C), electric potential (-5V), height below reference level

(c) FALSE - Scalar quantities can have dimensions

- Examples: Mass (kg), time (s), temperature (K), energy (J)
- Some are dimensionless: coefficient of friction, refractive index

(d) FALSE - Scalar quantities can vary in space

- Examples: Temperature varies from point to point, pressure varies with altitude
- Density varies in non-uniform materials

(e) TRUE - Scalar values don't depend on coordinate system orientation

- Mass, temperature, time, energy remain same regardless of axes choice
- This is a fundamental property distinguishing scalars from vectors

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3.22 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

Answer: Given:

- Height h = 3400 m
- Time interval Δt = 10.0 s
- Angle subtended θ = 30°

Analysis: The aircraft moves in an arc. From ground observation point, the aircraft appears to move through angle θ in time Δt.

Arc length calculation: The aircraft flies along arc of radius r (distance from observer to aircraft) $r = \sqrt{(h^2 + d^2)}$ where d is horizontal distance

For small angles or when height >> horizontal displacement: Arc length ≈ $r \times \theta$ (in radians) $\theta = 30° = 30 \times \pi/180 = \pi/6$ radians

Distance from observer to aircraft: $r \approx h = 3400$ m (assuming aircraft moves roughly perpendicular to line of sight)

Arc length traveled: $s = r \times \theta = 3400 \times \pi/6 = 3400\pi/6$ m

Speed of aircraft: $v = s/\Delta t = (3400\pi/6)/10.0 = 340\pi/6 =$ **178 m/s ≈ 641 km/h**

Source: NCERT Physics Class 11, Chapter 3 - Motion in a Plane