

Chapter 8: Mechanical Properties of Solids - NCERT Exercise Solutions

Physics Class 11

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8.1 Young's Modulus Ratio: Steel vs Copper

Given: Steel wire ($L_1 = 4.7$ m, $A_1 = 3.0 \times 10^{-5}$ m²) and copper wire ($L_2 = 3.5$ m, $A_2 = 4.0 \times 10^{-5}$ m²) stretch by same amount under same load.

Solution: For same load F and same elongation ΔL :

$$\Delta L = FL_1/(Y_1 A_1) = FL_2/(Y_2 A_2)$$

$$Y_1/Y_2 = (L_1/A_1)/(L_2/A_2) = (L_1 A_2)/(L_2 A_1)$$

$$Y_1/Y_2 = (4.7 \times 4.0 \times 10^{-5})/(3.5 \times 3.0 \times 10^{-5}) = 1.79$$

Answer: $Y_{\text{steel}}/Y_{\text{copper}} = 1.8$

8.2 Material Properties from Graph

From the stress-strain graph:

(a) Young's modulus:

$$Y = \text{stress/strain} = 150 \times 10^6 \text{ Pa} / 0.002 = 7.5 \times 10^{10} \text{ Pa}$$

(b) Approximate yield strength: Answer: $\sim 300 \times 10^6$ Pa (where linear relationship ends)

8.3 Comparing Materials A and B

From stress-strain graphs:

(a) Greater Young's modulus: Material A has **steeper slope** in linear region **(b) Stronger material:** Material B has **higher ultimate strength** (can withstand more stress before breaking)

8.4 True/False Statements

(a) Young's modulus of rubber > steel: FALSE - Steel has much higher Young's modulus (~ 200 GPa vs ~ 0.01 GPa for rubber)

(b) Coil stretching determined by shear modulus: TRUE - Coil springs undergo torsional deformation, which depends on shear modulus

8.5 Wire Elongations Under Load

Given: Steel wire ($d = 0.25$ cm, $L = 1.5$ m) and brass wire ($L = 1.0$ m) with 4.0 kg load

Solution:

$$\text{Area } A = \pi(0.125 \times 10^{-2})^2 = 4.91 \times 10^{-6} \text{ m}^2$$

$$\text{Stress} = F/A = (4.0 \times 9.8)/4.91 \times 10^{-6} = 8.0 \times 10^6 \text{ Pa}$$

$$\text{Steel elongation: } \Delta L_1 = (\text{stress} \times L)/Y = (8.0 \times 10^6 \times 1.5)/(2.0 \times 10^{11}) = 6 \times 10^{-5} \text{ m}$$

$$\text{Brass elongation: } \Delta L_2 = (8.0 \times 10^6 \times 1.0)/(0.91 \times 10^{11}) = 8.8 \times 10^{-5} \text{ m}$$

Answer: Steel = 0.06 mm, Brass = 0.088 mm

8.6 Aluminum Cube Shear Deflection

Given: Cube edge = 10 cm, mass = 100 kg attached to opposite face, $G = 25 \text{ GPa}$

Solution:

$$\text{Shear stress} = F/A = (100 \times 9.8)/(0.1)^2 = 9,800 \text{ Pa}$$

$$\text{Shear strain} = \text{stress}/G = 9,800/(25 \times 10^9) = 3.92 \times 10^{-7}$$

$$\text{Deflection} = \text{strain} \times \text{height} = 3.92 \times 10^{-7} \times 0.1 = 3.92 \times 10^{-8} \text{ m}$$

Answer: Vertical deflection = 39.2 nm

8.7 Steel Column Compression

Given: Four columns support 50,000 kg, inner radius = 30 cm, outer radius = 60 cm

Solution:

$$\text{Load per column} = 50,000/4 = 12,500 \text{ kg}$$

$$\text{Area per column} = \pi(0.6^2 - 0.3^2) = \pi(0.27) = 0.848 \text{ m}^2$$

$$\text{Stress} = (12,500 \times 9.8)/0.848 = 1.44 \times 10^5 \text{ Pa}$$

$$\text{Strain} = \text{stress}/Y = 1.44 \times 10^5/(2.0 \times 10^{11}) = 7.2 \times 10^{-7}$$

Answer: Compressional strain = 7.2×10^{-7}

8.8 Copper Tensile Strain

Given: Cross-section $15.2 \times 19.1 \text{ mm}^2$, force = 44,500 N, elastic deformation only

Solution:

$$\text{Area} = 15.2 \times 19.1 \times 10^{-6} = 2.90 \times 10^{-4} \text{ m}^2$$

$$\text{Stress} = F/A = 44,500 / (2.90 \times 10^{-4}) = 1.53 \times 10^8 \text{ Pa}$$

$$\text{Strain} = \text{stress}/Y = 1.53 \times 10^8 / (1.1 \times 10^{11}) = 1.39 \times 10^{-3}$$

Answer: Strain = 1.4×10^{-3} or 0.14%

8.9 Steel Cable Maximum Load

Given: Radius = 1.5 cm, maximum stress = 10^8 N/m^2

Solution:

$$\text{Area} = \pi r^2 = \pi (0.015)^2 = 7.07 \times 10^{-4} \text{ m}^2$$

$$\text{Maximum load} = \text{stress} \times \text{area} = 10^8 \times 7.07 \times 10^{-4} = 70,700 \text{ N}$$

Answer: Maximum load = 70.7 kN

8.10 Three-Wire Support System

Given: Steel bar (15 kg) supported by 3 wires (2 copper, 1 iron), each 2.0 m long, same tension

Solution: For same tension and same elongation:

$$\Delta L = FL/(YA) = \text{constant}$$

Therefore: $A \propto 1/Y$

$$A_{\text{copper}}/A_{\text{iron}} = Y_{\text{iron}}/Y_{\text{copper}} = (2.0 \times 10^{11})/(1.1 \times 10^{11}) = 1.82$$

$$d_{\text{copper}}/d_{\text{iron}} = \sqrt{1.82} = 1.35$$

Answer: Diameter ratio = 1.35:1

8.11 Whirling Mass Wire Extension

Given: $m = 14.5 \text{ kg}$, $L = 1.0 \text{ m}$, $\omega = 2 \text{ rev/s} = 4\pi \text{ rad/s}$, $A = 0.065 \text{ cm}^2$

Solution: At bottom of circle, total tension:

$$T = mg + m\omega^2 L = 14.5 \times 9.8 + 14.5 \times (4\pi)^2 \times 1.0 = 142 + 2,280 = 2,422 \text{ N}$$

$$\text{Stress} = T/A = 2,422/(0.065 \times 10^{-4}) = 3.73 \times 10^8 \text{ Pa}$$

$$\text{Extension} = (\text{stress} \times L)/Y = (3.73 \times 10^8 \times 1.0)/(2.0 \times 10^{11}) = 1.86 \times 10^{-3} \text{ m}$$

Answer: Extension = 1.86 mm

8.12 Water Bulk Modulus

Given: Initial volume = 100.0 L, pressure increase = 100.0 atm, final volume = 100.5 L

Solution:

$$\Delta V = 100.5 - 100.0 = 0.5 \text{ L}$$

$$\text{Volume strain} = \Delta V/V = 0.5/100.0 = 0.005$$

$$\text{Pressure} = 100 \times 1.013 \times 10^5 = 1.013 \times 10^7 \text{ Pa}$$

$$\text{Bulk modulus } B = \text{pressure}/\text{volume strain} = 1.013 \times 10^7 / 0.005 = 2.03 \times 10^9 \text{ Pa}$$

Air bulk modulus $\approx 10^5$ Pa Ratio: $B_{\text{water}}/B_{\text{air}} \approx 20,000$ (water much less compressible)

8.13 Water Density at Depth

Given: Pressure = 80.0 atm, surface density = $1.03 \times 10^3 \text{ kg/m}^3$

Solution:

$$\text{Pressure} = 80 \times 1.013 \times 10^5 = 8.1 \times 10^6 \text{ Pa}$$

$$\text{Volume strain} = \text{pressure}/B = 8.1 \times 10^6 / (2.2 \times 10^9) = 3.68 \times 10^{-3}$$

$$\text{Density increase} = \rho_0 \times \text{volume strain} = 1.03 \times 10^3 \times 3.68 \times 10^{-3} = 3.79 \text{ kg/m}^3$$

$$\text{Final density} = 1.03 \times 10^3 + 3.79 = 1.034 \times 10^3 \text{ kg/m}^3$$

Answer: Density = $1.034 \times 10^3 \text{ kg/m}^3$

8.14 Glass Volume Change

Given: Hydraulic pressure = 10 atm, $B_{\text{glass}} = 37 \times 10^9 \text{ Pa}$

Solution:

$$\text{Pressure} = 10 \times 1.013 \times 10^5 = 1.013 \times 10^6 \text{ Pa}$$

$$\text{Fractional volume change} = \text{pressure}/B = 1.013 \times 10^6 / (37 \times 10^9) = 2.74 \times 10^{-5}$$

Answer: $\Delta V/V = 2.7 \times 10^{-5}$ or 0.003%

8.15 Copper Cube Volume Contraction

Given: Edge = 10 cm, pressure = 7.0×10^6 Pa, $B_{\text{copper}} = 140 \times 10^9$ Pa

Solution:

Original volume = $(0.1)^3 = 10^{-3} \text{ m}^3$

Volume strain = pressure/ $B = 7.0 \times 10^6 / (140 \times 10^9) = 5 \times 10^{-5}$

Volume contraction = volume strain $\times$ original volume = $5 \times 10^{-5} \times 10^{-3} = 5 \times 10^{-8} \text{ m}^3$

Answer: Volume contraction = $5 \times 10^{-8} \text{ m}^3 = 0.05 \text{ cm}^3$

8.16 Water Pressure for 0.10% Compression

Given: Volume reduction = 0.10% = 10^{-3} , $B_{\text{water}} = 2.2 \times 10^9$ Pa

Solution:

Required pressure = $B \times \text{volume strain} = 2.2 \times 10^9 \times 10^{-3} = 2.2 \times 10^6$ Pa

Pressure increase = 2.2×10^6 Pa = 21.7 atm

Answer: Pressure increase = 2.2 MPa or 22 atm

Key Formulas Reference

Stress and Strain

- **Stress:** $\sigma = F/A$
- **Longitudinal strain:** $\varepsilon = \Delta L/L$

- **Volume strain:** $\Delta V/V$
- **Shear strain:** $\gamma = \Delta x/L = \tan \theta$

Elastic Moduli

- **Young's modulus:** $Y = \sigma/\epsilon = (F/A)/(\Delta L/L)$
- **Shear modulus:** $G = \tau/\gamma = (F/A)/\theta$
- **Bulk modulus:** $B = -p/(\Delta V/V)$

Key Relationships

- **Hooke's law:** Stress $\propto$ Strain (within elastic limit)
- **Energy density:** $u = \frac{1}{2}\sigma\epsilon = \frac{1}{2}(\text{stress} \times \text{strain})$
- **Poisson's ratio:** $\nu = (\text{lateral strain})/(\text{longitudinal strain})$

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