NCERT Class 11 Physics

Chapter 12: Kinetic Theory

Questions and Answers

BACK EXERCISES (12.1 - 12.10)

Question 12.1

Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3 Å.

Answer: Given:

- Diameter of O_2 molecule, $d = 3$ Å $= 3 \times 10^{-10}$ m
- At STP: P = 1 atm, T = 273 K

Step 1: Volume of one O_2 molecule V_molecule = $(4/3)\pi r^3 = (4/3)\pi(1.5 \times 10^{-10})^3$ V_molecule = 1.41×10^{-29} m^3

Step 2: Number of molecules in 1 mole at STP N = 6.02×10^{23} molecules

Step 3: Total molecular volume in 1 mole V_molecular_total = N × V_molecule V_molecular_total = $6.02 \times 10^{23} \times 1.41 \times 10^{-29} = 8.5 \times 10^{-6}$ m^3

Step 4: Molar volume at STP V_molar = 22.4 L = 2.24×10^{-2} m^3

Step 5: Fraction Fraction = V_molecular_total / V_molar Fraction = $(8.5 \times 10^{-6}) / (2.24 \times 10^{-2}) = 3.8 \times 10^{-4}$

Answer: 3.8×10^{-4}

Question 12.2

Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP: 1 atmospheric pressure, 0°C). Show that it is 22.4 litres.

Answer: Given:

- STP conditions: $P = 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$, $T = 273\ \text{K}$
- $n = 1$ mole
- $R = 8.314\ \text{J mol}^{-1}\ \text{K}^{-1}$

Using ideal gas equation: $PV = nRT$

$V = nRT/P = (1\ \text{mol} \times 8.314\ \text{J mol}^{-1}\ \text{K}^{-1} \times 273\ \text{K}) / (1.013 \times 10^5\ \text{Pa})$

$V = 2269.7 / (1.013 \times 10^5) = 0.0224\ \text{m}^3 = 22.4\ \text{L}$

Answer: Proved that molar volume = 22.4 L at STP

Question 12.3

Figure 12.8 shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.

(a) What does the dotted plot signify?
(b) Which is true: $T_1 > T_2$ or $T_1 < T_2$?
(c) What is the value of PV/T where the curves meet on the y-axis?
(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T?

Answer:

(a) The dotted plot signifies the ideal gas behavior (PV/T = constant for all pressures).

(b) $T_1 > T_2$ Since at the same pressure, the gas with higher temperature has higher PV/T value.

(c) At y-axis intersection ($P \rightarrow 0$), the gas behaves ideally. For oxygen: PV/T = μR where μ = number of moles μ = mass/molar mass = $(1.00 \times 10^{-3}$ kg$)/(32 \times 10^{-3}$ kg/mol$) = 0.03125$ mol PV/T = $0.03125 \times 8.314 = 0.26$ J K^{-1}

(d) For hydrogen: $\mu\_H_2 = (1.00 \times 10^{-3})/(2.02 \times 10^{-3}) = 0.495$ mol PV/T = $0.495 \times 8.314 = 4.12$ J K^{-1}

To get same PV/T value as oxygen:
Required moles = 0.03125 mol
Mass of H_2 needed = $0.03125 \times 2.02 \times 10^{-3} = 6.31 \times 10^{-5}$ kg

Answers: (a) Ideal gas behavior (b) $T_1 > T_2$ (c) 0.26 J K^{-1} (d) 6.31×10^{-5} kg

---

Question 12.4

An oxygen cylinder of volume 30 litre has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.

Answer: Given:

- V = 30 L = 0.03 m^3
- Initial: $P_1 = 15$ atm $= 15.2 \times 10^5$ Pa, $T_1 = 300$ K
- Final: $P_2 = 11$ atm $= 11.1 \times 10^5$ Pa, $T_2 = 290$ K
- Molar mass of O_2 = 32 g/mol = 0.032 kg/mol

Initial number of moles: $n_1 = P_1V/(RT_1) = (15.2 \times 10^5 \times 0.03)/(8.314 \times 300) = 18.3$ mol

Final number of moles: $n_2 = P_2V/(RT_2) = (11.1 \times 10^5 \times 0.03)/(8.314 \times 290) = 13.8$ mol

Moles withdrawn: $\Delta n = n_1 - n_2 = 18.3 - 13.8 = 4.5$ mol

Mass withdrawn: $\Delta m = \Delta n \times M = 4.5 \times 0.032 = 0.144$ kg

Answer: 0.144 kg or 144 g

Question 12.5

An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C?

Answer: Given:

- Initial: $V_1 = 1.0$ cm^3, depth = 40 m, $T_1 = 285$ K
- Final: $T_2 = 308$ K, at surface

Initial pressure: $P_1 = P\_atm + \rho gh = 1 \text{ atm} + (40 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2)/(1.013 \times 10^5 \text{ Pa/atm})$
$P_1 = 1 + 3.87 = 4.87$ atm

Final pressure: $P_2 = 1$ atm

Using combined gas law: $(P_1V_1)/T_1 = (P_2V_2)/T_2$

$V_2 = (P_1V_1T_2)/(P_2T_1) = (4.87 \times 1.0 \times 308)/(1 \times 285) = 5.26$ cm^3

Answer: 5.3 cm^3

Question 12.6

Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27°C and 1 atm pressure.

Answer: Given:

- $V = 25.0\ m^3$
- $T = 300\ K$
- $P = 1\ atm = 1.013 \times 10^5\ Pa$

Using ideal gas equation: $PV = Nk_\beta T$

$N = PV/(k_\beta T) = (1.013 \times 10^5 \times 25.0)/(1.38 \times 10^{-23} \times 300)$

$N = 2.533 \times 10^6 / (4.14 \times 10^{-21}) = 6.12 \times 10^{26}$ molecules

Answer: 6.1×10^{26} molecules

Question 12.7

Estimate the average thermal energy of a helium atom at (i) room temperature (27°C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).

Answer: For a monatomic gas, average thermal energy per atom = $(3/2)k_\beta T$

(i) At room temperature ($T = 300$ K): $E = (3/2) \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}$ J

(ii) At Sun's surface ($T = 6000$ K): $E = (3/2) \times 1.38 \times 10^{-23} \times 6000 = 1.24 \times 10^{-19}$ J

(iii) At stellar core ($T = 10^7$ K): $E = (3/2) \times 1.38 \times 10^{-23} \times 10^7 = 2.07 \times 10^{-16}$ J

Answers: (i) 6.2×10^{-21} J (ii) 1.2×10^{-19} J (iii) 2.1×10^{-16} J

Question 12.8

Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Answer:

Part 1: Equal number of molecules Yes, according to Avogadro's hypothesis, equal volumes of gases at the same temperature and pressure contain equal number of molecules.

Part 2: RMS speeds No, RMS speeds are not the same.

$v_{rms} = \sqrt{(3k_\beta T/m)} = \sqrt{(3RT/M)}$

Since temperature is same, $v_{rms} \propto 1/\sqrt{M}$

Molecular masses:

- Neon (Ne): 20.2 u
- Chlorine (Cl_2): 70.9 u
- Uranium hexafluoride (UF_6): 238 + 6×19 = 352 u

Order of RMS speeds: $v_{ne} > vCl_2 > vUF_6$

Answer: Yes, equal number of molecules. No, RMS speeds different. Neon has largest v_{rms}.

Question 12.9

At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at –20°C?

Answer: Given:

- Helium at $T_1 = 253$ K
- Atomic masses: Ar = 39.9 u, He = 4.0 u

For equal RMS speeds: $v_{rms}(Ar) = v_{rms}(He)$

$\sqrt{(3RT_2/M\_Ar)} = \sqrt{(3RT_1/M\_He)}$

$\sqrt{(T_2/M\_Ar)} = \sqrt{(T_1/M\_He)}$

$T_2/M\_Ar = T_1/M\_He$

$T_2 = T_1 \times M\_Ar/M\_He = 253 \times 39.9/4.0 = 2524$ K

Answer: 2524 K or 2251°C

Question 12.10

Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions.

Answer: Given:

- $P = 2.0$ atm $= 2.026 \times 10^5$ Pa
- $T = 290$ K
- Radius $r = 1.0$ Å, so diameter $d = 2.0 \times 10^{-10}$ m
- Molecular mass of $N_2 = 28.0$ u $= 4.65 \times 10^{-26}$ kg

Step 1: Number density $n = P/(k_\beta T) = (2.026 \times 10^5)/(1.38 \times 10^{-23} \times 290) = 5.06 \times 10^{25}\ m^{-3}$

Step 2: Mean free path $l = 1/(\sqrt{2} \times n \times \pi \times d^2)$ $l = 1/(\sqrt{2} \times 5.06 \times 10^{25} \times \pi \times (2.0 \times 10^{-10})^2)$ $l = 1/(8.97 \times 10^{14}) = 1.11 \times 10^{-7}$ m

Step 3: Average speed $\langle v \rangle = \sqrt{(8k_\beta T/(\pi m))} = \sqrt{(8 \times 1.38 \times 10^{-23} \times 290/(\pi \times 4.65 \times 10^{-26}))}$ $\langle v \rangle = 471$ m/s

Step 4: Collision frequency $\nu = \langle v \rangle/l = 471/(1.11 \times 10^{-7}) = 4.24 \times 10^9\ s^{-1}$

Step 5: Time between collisions $\tau = 1/\nu = 2.36 \times 10^{-10}$ s

Step 6: Collision time estimate Collision time $\approx d/\langle v \rangle = (2.0 \times 10^{-10})/471 = 4.24 \times 10^{-13}$ s

Comparison: Time between collisions is about 500 times larger than collision time.

Answers:

- Mean free path: 1.11×10^{-7} m
- Collision frequency: $4.24 \times 10^9\ s^{-1}$
- Collision time << Time between collisions (by factor ~500)

POINTS TO PONDER - DISCUSSION QUESTIONS

Point 1

Pressure of a fluid is not only exerted on the wall. Pressure exists everywhere in a fluid. Any layer of gas inside the volume of a container is in equilibrium because the pressure is the same on both sides of the layer.

Discussion: This emphasizes that pressure is a property of the fluid at every point, not just at boundaries. Pascal's principle relies on this uniform pressure distribution.

Point 2

We should not have an exaggerated idea of the intermolecular distance in a gas. At ordinary pressures and temperatures, this is only 10 times or so the interatomic distance in solids and liquids. What is different is the mean free path which in a gas is 100 times the interatomic distance and 1000 times the size of the molecule.

Discussion: The key difference between gases and condensed phases is not just molecular spacing, but the ability to travel long distances without collision (mean free path).

Point 3

The law of equipartition of energy is stated thus: the energy for each degree of freedom in thermal equilibrium is $\frac{1}{2}k_\beta T$. Each quadratic term in the total energy expression of a molecule is to be counted as a degree of freedom. Thus, each vibrational mode gives 2 (not 1) degrees of freedom (kinetic and potential energy modes), corresponding to the energy $2 \times \frac{1}{2}k_\beta T = k_\beta T$.

Discussion: This clarifies why vibrational modes contribute twice as much energy as translational or rotational modes.

Point 4

Molecules of air in a room do not all fall and settle on the ground (due to gravity) because of their high speeds and incessant collisions. In equilibrium, there is a very slight increase in density at lower heights (like in the atmosphere). The effect is small since the potential energy (mgh) for ordinary heights is much less than the average kinetic energy $\frac{1}{2}mv^2$ of the molecules.

Discussion: Thermal motion dominates over gravitational effects for gas molecules at ordinary conditions. This explains why gases uniformly fill containers.

Point 5

$\langle v^2 \rangle$ is not always equal to $(\langle v \rangle)^2$. The average of a squared quantity is not necessarily the square of the average.

Discussion: This is a fundamental statistical concept. For example, if velocities are +1 and -1, then $\langle v \rangle = 0$ but $\langle v^2 \rangle = 1$. This distinction is crucial for understanding RMS values versus arithmetic mean values.

ADDITIONAL CONCEPTUAL QUESTIONS

Q1: Why does kinetic theory work better for gases than for solids or liquids?

Answer: In gases, intermolecular forces are negligible except during brief collisions, making the independent particle model valid. In solids and liquids, continuous intermolecular interactions dominate the behavior.

Q2: Explain why all gases have the same molar volume at STP despite having different molecular sizes.

Answer: At STP, molecular volumes are negligible compared to the space between molecules. The ideal gas behavior depends on the number of molecules, not their individual sizes.

Q3: How does the equipartition theorem explain the temperature independence of specific heats for ideal gases?

Answer: Each degree of freedom contributes a fixed amount ($\frac{1}{2}k_\beta T$) to the average energy. Since the number of degrees of freedom is constant for a given molecule type, the specific heat becomes independent of temperature.

Q4: Why is the mean free path much larger than molecular dimensions?

Answer: The mean free path depends on the collision cross-section and number density. Even though molecules have finite size, the probability of collision over short distances is small due to the three-dimensional random motion and relatively low density of gases.

End of Questions and Answers

Total Questions Covered:

- Back Exercises: 12.1 to 12.10 (10 questions)
- Points to Ponder: 5 discussion points
- Additional Conceptual Questions: 4 questions

Note: All numerical answers are rounded to appropriate significant figures based on given data precision.