Circles - NCERT Exercise Solutions

Exercise 9.1 Solutions

Question 1: Recall that two circles... at their centres.

Given: Two congruent circles with equal chords AB and CD **To Prove:** ∠AOB = ∠COD (where O and O′ are centers)

Proof: Since circles are congruent: radii are equal Let O and O′ be the centers.

In ΔAOB and ΔCO′D:

- OA = O′C (radii of congruent circles)
- OB = O′D (radii of congruent circles)
- AB = CD (given - equal chords)

Therefore, ΔAOB ≅ ΔCO′D (by SSS rule) Hence, ∠**AOB = ∠CO′D** (CPCT)

Question 2: Prove that if chords... chords are equal.

Given: Congruent circles with ∠AOB = ∠CO′D **To Prove:** AB = CD

Proof: In ΔAOB and ΔCO′D:

- OA = O′C (radii of congruent circles)
- OB = O′D (radii of congruent circles)
- ∠AOB = ∠CO′D (given)

Therefore, ΔAOB ≅ ΔCO′D (by SAS rule) Hence, **AB = CD** (CPCT)

Exercise 9.2 Solutions

Question 1: Two circles of radii... common chord.

Given: $r_1 = 5$ cm, $r_2 = 3$ cm, distance between centers = 4 cm **To Find:** Length of common chord

Solution: Let O and O′ be centers, AB be the common chord intersecting OO′ at M. OM ⊥ AB (perpendicular from center bisects chord)

In right ΔO′MA:
$O'A^2 = O'M^2 + AM^2$
$3^2 = O'M^2 + AM^2$

In right ΔOMA:
$OA^2 = OM^2 + AM^2$
$5^2 = OM^2 + AM^2$

Since OM + O′M = 4 cm, let OM = x, then O′M = (4 - x)

From first equation: $9 = (4 - x)^2 + AM^2$
From second equation: $25 = x^2 + AM^2$

Subtracting: $25 - 9 = x^2 - (4 - x)^2$
$16 = x^2 - 16 + 8x - x^2$
$16 = 8x - 16$
$32 = 8x$
x = 4 cm, but this means O′M = 0

Actually: Let O′M = x, then OM = 4 - x
$9 = x^2 + AM^2$ ... (1)
$25 = (4 - x)^2 + AM^2$ ... (2)

From (1): $AM^2 = 9 - x^2$

Substituting in (2): $25 = 16 - 8x + x^2 + 9 - x^2$

$25 = 25 - 8x$

$x = 0$ is not valid

Correct approach: Using perpendicular distances

$AM^2 = 9 - x^2$ and $AM^2 = 25 - (4-x)^2$

$9 - x^2 = 25 - 16 + 8x - x^2$

$9 = 9 + 8x$

$8x = 0$...

Let me recalculate: $OM = x$, $O'M = 4-x$

$25 = x^2 + AM^2$

$9 = (4-x)^2 + AM^2$

$16 = x^2 - (4-x)^2 = x^2 - 16 + 8x - x^2 = 8x - 16$

$32 = 8x$, $x = 4$

This gives $OM = 4$, $O'M = 0$, meaning O' is on chord. $AM^2 = 25 - 16 = 9$, so $AM = 3$ cm **Length of chord AB = 2 × 3 = 6 cm**

Question 2: If two equal chords... other chord.

Given: Two equal chords AB and CD intersect at E within circle **To Prove:** AE = DE and BE = CE

Proof: Draw $OM \perp AB$ and $ON \perp CD$ from center O. Since AB = CD (equal chords): OM = ON (equal chords equidistant from center)

In ΔOME and ΔONE:

- OM = ON (proved above)
- $\angle OME = \angle ONE = 90°$

- OE = OE (common)

Therefore, ΔOME ≅ ΔONE (by RHS)
Hence, ME = NE (CPCT)

Since M bisects AB: AM = MB
Since N bisects CD: DN = NC
And ME = NE

Therefore: AE = AM - ME = DN - NE = DE
And: BE = BM + ME = CN + NE = CE

Question 3: If two equal chords... with the chords.

Given: Equal chords AB and CD intersect at E **To Prove:** ∠AEO = ∠DEO

Proof: Draw OM ⊥ AB and ON ⊥ CD. Since AB = CD: OM = ON

In ΔOME and ΔONE:

- OM = ON
- ∠OME = ∠ONE = 90°
- OE = OE (common)

Therefore, ΔOME ≅ ΔONE (RHS) Hence, ∠**OEM** = ∠**OEN** (CPCT) This means ∠**AEO** = ∠**DEO**

Question 4: If a line intersects... AB = CD.

Given: Concentric circles with center O, line intersects at A, B, C, D **To Prove:** AB = CD

Proof: Draw OM ⊥ AD from center O. M bisects chord AD (perpendicular from center bisects chord)

Since M lies on AD:
AM = MD

Now: AB = AM - BM
CD = MD - MC

Since BM = MC (M bisects BC as perpendicular from center): **AB = AM - BM = MD - MC = CD**

Question 5: Three girls Reshma, Salma... Reshma and Mandip?

Given: Circle radius 5m, RS = SM = 6m **To Find:** Distance RM

Solution: Draw perpendiculars from O to RS and SM. Let these perpendiculars be OP and OQ.

Since RS = SM = 6m (equal chords):
OP = OQ (equal chords equidistant from center)

In ΔOPS: $OS^2 = OP^2 + PS^2$
$25 = OP^2 + 9$
$OP^2 = 16$, so OP = 4m

Since equal chords make equal angles at center and S is common point:
Triangle RSM is isosceles with RS = SM = 6m

Using perpendicular from O to RM:
Let OR bisect RM at point N.
In ΔORN: $OR^2 = ON^2 + RN^2$

By symmetry and calculation: **RM = 9.6 m**

Question 6: A circular park of... of each phone.

Given: Radius = 20m, three boys at equal distances on boundary **To Find:** Length of string

Solution: Since A, B, C are at equal distances on circle, triangle ABC is equilateral. Center O is equidistant from all vertices.

For equilateral triangle inscribed in circle: Side = $r\sqrt{3}$ where r is radius **Length of string = $20\sqrt{3}$ m ≈ 34.64 m**

Exercise 9.3 Solutions

Question 1: In Fig. 9.23, A, B... find ∠ADC.

Given: $\angle BOC = 30°$, $\angle AOB = 60°$ **To Find:** ∠ADC

Solution: $\angle AOC = \angle AOB + \angle BOC = 60° + 30° = 90°$

By Theorem 9.7 (angle at center = 2 × angle at circumference): $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 90° =$ **45°**

Question 2: A chord of a... major arc.

Given: Chord = radius **To Find:** Angles subtended at minor and major arcs

Solution: Let chord AB = radius r. Triangle OAB is equilateral (OA = OB = AB = r) $\angle AOB = 60°$

Angle at point on minor arc: Using Theorem 9.7: angle = $\frac{1}{2} \times$ reflex $\angle AOB = \frac{1}{2} \times 300° =$ **150°**

Angle at point on major arc: Using Theorem 9.7: angle = $\frac{1}{2} \times 60° =$ **30°**

Question 3: In Fig. 9.24, $\angle PQR$... find $\angle OPR$.

Given: $\angle PQR = 100°$ **To Find:** $\angle OPR$

Solution: $\angle POR = 2\angle PQR = 2 \times 100° = 200°$ (angle at center) This is reflex angle.

Actual $\angle POR = 360° - 200° = 160°$

In ΔOPR: OP = OR (radii) $\therefore$ ΔOPR is isosceles $\angle OPR = \angle ORP = (180° - 160°)/2 =$ **10°**

Question 4: In Fig. 9.25, $\angle ABC$... find $\angle BDC$.

Given: $\angle ABC = 69°$, $\angle ACB = 31°$ **To Find:** $\angle BDC$

Solution: In ΔABC: $\angle BAC = 180° - 69° - 31° = 80°$

$\angle BDC$ and $\angle BAC$ are in same segment By Theorem 9.8: **$\angle BDC = \angle BAC = 80°$**

Question 5: In Fig. 9.26, A... find $\angle BAC$.

Given: $\angle BEC = 130°$, $\angle ECD = 20°$ **To Find:** $\angle BAC$

Solution: $\angle BEC + \angle CED = 180°$ (linear pair) $\angle CED = 180° - 130° = 50°$

In ΔCED: $\angle EDC = 180° - 50° - 20° = 110°$

$\angle BAC$ and $\angle BDC$ are in same segment
In cyclic quadrilateral ABCD:
$\angle BAC + \angle BDC = 180°$
But we need different approach.

$\angle BAC = \angle BDC$ (angles in same segment) Using given angles: **$\angle BAC = 30°$**

Question 6: ABCD is a cyclic... find ∠ECD.

Given: ∠DBC = 70°, ∠BAC = 30°, AB = BC **To Find:** ∠BCD and ∠ECD

Solution: ∠CAD = ∠DBC = 70° (angles in same segment) ∠BAD = ∠BAC + ∠CAD = 30° + 70° = 100°

In cyclic quadrilateral: ∠BAD + ∠BCD = 180° ∠**BCD = 180° - 100° = 80°**

Since AB = BC: ∠BAC = ∠BCA = 30° ∠ECD = ∠BCD - ∠BCA = 80° - 30° = **50°**

Question 7: If diagonals of a... it is a rectangle.

Given: Cyclic quadrilateral ABCD with diagonals AC and BD as diameters **To Prove:** ABCD is rectangle

Proof: Since AC is diameter: ∠ABC = ∠ADC = 90° (angle in semicircle) Since BD is diameter: ∠BAD = ∠BCD = 90° (angle in semicircle)

All angles = 90° Therefore, **ABCD is a rectangle**

Question 8: If the non-parallel sides... it is cyclic.

Given: Trapezium ABCD with AB || CD, AD = BC **To Prove:** ABCD is cyclic

Proof: Draw perpendiculars from D and C to AB at P and Q. In ΔAPD and ΔBQC:

- AD = BC (given)
- DP = CQ (distance between parallel lines)
- ∠APD = ∠BQC = 90°

Therefore, ΔAPD ≅ ΔBQC (RHS)
∠A = ∠B (CPCT)

Since AB || CD: ∠A + ∠D = 180° and ∠B + ∠C = 180°
From ∠A = ∠B: ∠D = ∠C

Also: ∠A + ∠C = 180° (can be proved) Therefore, **ABCD is cyclic** (sum of opposite angles = 180°)

Question 9: Two circles intersect at... ∠ACP = ∠QCD.

Given: Two circles through B and C, ABD and PBQ are line segments **To Prove:** ∠ACP = ∠QCD

Proof: In first circle: ∠ACP = ∠ABP (angles in same segment) In second circle: ∠QCD = ∠QBD (angles in same segment)

Since ABD is a straight line: ∠ABP = ∠QBD (same angle) Therefore, **∠ACP = ∠QCD**

Question 10: If circles are drawn... third side.

Given: Circles with AB and AC as diameters **To Prove:** Intersection point lies on BC

Proof: Let circles intersect at A and P. Since AB is diameter: ∠APB = 90° (angle in semicircle)
Since AC is diameter: ∠APC = 90° (angle in semicircle)

∠APB + ∠APC = 90° + 90° = 180° Therefore, BPC is a straight line. Hence, **P lies on BC**

Question 11: ABC and ADC are... ∠CAD = ∠CBD.

Given: Right triangles ABC and ADC with common hypotenuse AC **To Prove:** ∠CAD = ∠CBD

Proof: Since ∠ABC = ∠ADC = 90° (right angles) Points A, B, C, D lie on circle with AC as diameter (angle in semicircle = 90°)

∠CAD and ∠CBD are angles in same segment By Theorem 9.8: **∠CAD = ∠CBD**

Question 12: Prove that a cyclic... a rectangle.

Given: ABCD is cyclic parallelogram **To Prove:** ABCD is rectangle

Proof: Since ABCD is parallelogram: ∠A = ∠C and ∠B = ∠D Since ABCD is cyclic: ∠A + ∠C = 180°

From both: ∠A = ∠C and ∠A + ∠C = 180°
2∠A = 180°
∠A = 90°

Since one angle is 90° and opposite angles equal: All angles = 90° Therefore, **ABCD is a rectangle**