Triangles - Formula Sheet

Chapter 7 - Class 9 Mathematics

PAGE 1: CONGRUENCE BASICS

Definition of Congruence

Two triangles are congruent if:

- All three sides are equal
- All three angles are equal
- One triangle can be superimposed on the other

Notation: $\Delta ABC \cong \Delta PQR$

Important Points

- **CPCT**: Corresponding Parts of Congruent Triangles are equal
- **Vertex Correspondence**: $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$
- Order of vertices matters: $\Delta ABC \cong \Delta PQR$ ✓, but $\Delta ABC \cong \Delta QPR$ ✗

Congruence Criteria (Rules)

1. SAS (Side-Angle-Side) - AXIOM

Two sides and the included angle

- $AB = PQ$

- AC = PR
- ∠A = ∠P **Then**: ΔABC ≅ ΔPQR

Key: Angle must be between the two sides

2. ASA (Angle-Side-Angle) - THEOREM

Two angles and the included side

- ∠B = ∠Q
- BC = QR
- ∠C = ∠R **Then**: ΔABC ≅ ΔPQR

Key: Side must be between the two angles

3. AAS (Angle-Angle-Side) - THEOREM

Two angles and a corresponding side

- ∠A = ∠P
- ∠B = ∠Q
- BC = QR (or any corresponding side) **Then**: ΔABC ≅ ΔPQR

Note: Third angle is automatically equal (angle sum = 180°)

4. SSS (Side-Side-Side) - THEOREM

All three sides equal

- AB = PQ
- BC = QR
- AC = PR **Then**: ΔABC ≅ ΔPQR

5. RHS (Right angle-Hypotenuse-Side) - THEOREM

For right triangles only

- ∠B = ∠Q = 90°
- AC = PR (hypotenuse)
- AB = PQ (one side) **Then**: ΔABC ≅ ΔPQR

Invalid Rules ✗

- **SSA or ASS**: NOT a valid congruence rule
- **AAA**: Only shows similarity, not congruence
- Angle must be included for SAS to work

PAGE 2: ISOSCELES TRIANGLE PROPERTIES

Definition

Isosceles Triangle: A triangle with two equal sides

Theorem 7.2: Angles Opposite to Equal Sides

If two sides of a triangle are equal, then angles opposite to them are equal

Formula: If AB = AC, then ∠B = ∠C

Proof Method: Draw angle bisector from A to BC, use SAS congruence

Theorem 7.3: Sides Opposite to Equal Angles (CONVERSE)

If two angles of a triangle are equal, then sides opposite to them are equal

Formula: If ∠B = ∠C, then AB = AC

Proof Method: Use ASA congruence rule

Equilateral Triangle

All sides equal: AB = BC = CA **All angles equal**: ∠A = ∠B = ∠C = 60°

Formula: Each angle of equilateral triangle = **60°**

Proof:

- All sides equal → all angles equal (Theorem 7.3)
- ∠A + ∠B + ∠C = 180°
- 3∠A = 180°
- ∠A = 60°

Important Results for Isosceles Triangles

Perpendicular from Vertex to Base

If ΔABC is isosceles with AB = AC, and AD ⊥ BC:

1. **AD bisects BC**: BD = DC
2. **AD bisects ∠A**: ∠BAD = ∠CAD
3. **D is midpoint of BC**

Angle Bisector in Isosceles Triangle

If AB = AC and AD bisects ∠A:

- AD ⊥ BC (perpendicular to base)
- BD = DC (bisects base)

Equal Altitudes

If two altitudes of a triangle are equal, then the triangle is isosceles

Formula: If BE = CF (altitudes), then AB = AC

PAGE 3: PROBLEM-SOLVING TECHNIQUES

Strategy 1: Identify Congruent Triangles

Steps:

1. Mark given equal sides/angles
2. Look for common sides/angles
3. Find vertically opposite angles
4. Check for alternate/corresponding angles (parallel lines)
5. Choose appropriate congruence rule

Strategy 2: Using Properties

For proving sides equal:

- Show triangles congruent → use CPCT

For proving angles equal:

- Show triangles congruent → use CPCT
- Use isosceles triangle property

For proving lines parallel:

- Show alternate angles equal

- Show corresponding angles equal

Common Triangle Configurations

Perpendicular Bisector

Point P on perpendicular bisector of AB:

- **PA = PB** (equidistant from endpoints)
- Use RHS congruence

Angle Bisector

Point P on angle bisector of ∠A, with PB ⊥ one arm, PC ⊥ other arm:

- **PB = PC** (equidistant from arms)
- Use AAS or ASA congruence

Median

Line from vertex to midpoint of opposite side

- Creates two triangles with equal bases
- Useful for proving congruence with additional conditions

Key Proof Patterns

Pattern 1: Show AB = CD

1. Identify triangles containing AB and CD
2. Prove those triangles congruent
3. Conclude AB = CD by CPCT

Pattern 2: Show ∠A = ∠B

1. Find triangles containing these angles
2. Prove triangles congruent
3. Conclude angles equal by CPCT

Pattern 3: Show Line Bisects Segment

1. Show two segments are equal
2. Use definition of midpoint/bisector

Pattern 4: Show Lines Perpendicular

1. Prove angles equal to 90°
2. Use linear pair or triangle properties

PAGE 4: QUICK REFERENCE & FORMULAS

Triangle Angle Sum

∠A + ∠B + ∠C = 180°

Special Triangles Summary

Triangle Type	Sides	Angles
Equilateral	All equal	Each 60°
Isosceles	Two equal	Two equal
Right-angled	-	One 90°
Scalene	All different	All different

Congruence Rules Quick Check

Rule	Need	Don't Confuse With
SAS	2 sides + included angle	SSA (invalid)
ASA	2 angles + included side	SAA
AAS	2 angles + non-included side	-
SSS	All 3 sides	-
RHS	Right angle + hypotenuse + side	Only for right triangles

When to Use Which Rule

Use SAS when:

- Two sides are given/proven equal
- Angle between them is equal
- Common side present

Use ASA when:

- Two angles are equal
- Common side between them
- Often with parallel lines (alternate angles)

Use AAS when:

- Two angles equal
- One side equal (not between angles)
- Third angle automatically equal

Use SSS when:

- All three sides can be shown equal
- Common side + two other equal sides

Use RHS when:

- Triangle is right-angled
- Hypotenuse given
- One other side given

Important Theorems Checklist

✓ **Theorem 7.1 (ASA)**: Two angles + included side → congruent ✓ **Theorem 7.2**: Equal sides → equal opposite angles ✓ **Theorem 7.3**: Equal angles → equal opposite sides
✓ **Theorem 7.4 (SSS)**: Three sides equal → congruent ✓ **Theorem 7.5 (RHS)**: Right angle + hypotenuse + side → congruent

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Common Proof Statements

Proving Congruence

"In ΔABC and ΔPQR:

- [side/angle] = [side/angle] (Given/Proved/Common)
- [side/angle] = [side/angle] (reason)
- [side/angle] = [side/angle] (reason) Therefore, ΔABC ≅ ΔPQR (by SAS/ASA/AAS/SSS/RHS) Hence, [required result] (by CPCT)"

Using Isosceles Property

"In ΔABC, AB = AC (given/proved)
Therefore, ∠B = ∠C (angles opposite equal sides)"

Using Angle Bisector

"AD bisects ∠A (given)
Therefore, ∠BAD = ∠CAD = ½∠A"

Problem-Solving Checklist

Before starting any proof:

- [ ] Mark all given information on diagram
- [ ] Identify what needs to be proved
- [ ] Look for congruent triangles
- [ ] Check for common sides/angles
- [ ] Identify which congruence rule applies
- [ ] Write proof systematically
- [ ] State CPCT for final conclusion

Common Mistakes to Avoid

❌ Using SSA/ASS rule (invalid)
❌ Wrong vertex correspondence in congruence
❌ Forgetting to state "common side"
❌ Not mentioning which congruence rule used
❌ Assuming angles equal without proof

❌ Using RHS for non-right triangles
❌ Not stating CPCT at the end

✅ Always check angle is included for SAS
✅ Verify all conditions before applying rule
✅ State reasons for each step
✅ Mark diagrams clearly
✅ Write correspondence correctly