Quadrilaterals - Formula Sheet

Chapter 8 - Class 9 Mathematics

PART 1: PARALLELOGRAM PROPERTIES

Definition

A quadrilateral with both pairs of opposite sides parallel

Notation: ABCD is a parallelogram means $AB \parallel DC$ and $AD \parallel BC$

Key Theorems

Theorem 8.1: Diagonal divides parallelogram into two congruent triangles

- If ABCD is parallelogram, then $\Delta ABC \cong \Delta CDA$

Theorem 8.2: Opposite sides are equal

- $AB = DC$ and $AD = BC$

Theorem 8.3 (Converse): If opposite sides are equal, then it's a parallelogram

- If $AB = DC$ and $AD = BC$, then ABCD is parallelogram

Theorem 8.4: Opposite angles are equal

- $\angle A = \angle C$ and $\angle B = \angle D$

Theorem 8.5 (Converse): If opposite angles are equal, then it's a parallelogram

- If $\angle A = \angle C$ and $\angle B = \angle D$, then ABCD is parallelogram

Theorem 8.6: Diagonals bisect each other

- If diagonals AC and BD meet at O, then OA = OC and OB = OD

Theorem 8.7 (Converse): If diagonals bisect each other, then it's a parallelogram

- If OA = OC and OB = OD, then ABCD is parallelogram

Adjacent Angles

∠A + ∠B = 180° (co-interior angles) **∠B + ∠C = 180° ∠C + ∠D = 180° ∠D + ∠A = 180°**

PART 2: SPECIAL QUADRILATERALS

Rectangle

Definition: Parallelogram with one angle 90°

Properties:

- All angles = 90°
- Opposite sides equal: AB = DC, AD = BC
- Diagonals equal: AC = BD
- Diagonals bisect each other

Test: Parallelogram with equal diagonals → Rectangle

Rhombus

Definition: Parallelogram with all sides equal

Properties:

- All sides equal: AB = BC = CD = DA

- Opposite angles equal
- Diagonals bisect each other at right angles: AC ⊥ BD
- Diagonals bisect opposite angles

Test: Parallelogram with perpendicular diagonals → Rhombus

Square

Definition: Rectangle with all sides equal (OR Rhombus with one angle 90°)

Properties:

- All sides equal: AB = BC = CD = DA
- All angles = 90°
- Diagonals equal: AC = BD
- Diagonals bisect at right angles: AC ⊥ BD
- Diagonals bisect angles (each 45°)

Test: Rectangle with all sides equal → Square

Trapezium

Definition: Quadrilateral with one pair of parallel sides

Properties:

- One pair parallel: AB || DC (but AD ∦ BC)
- If AD = BC (isosceles trapezium), then ∠A = ∠B and ∠C = ∠D

PART 3: TESTS FOR PARALLELOGRAMS

A quadrilateral is a parallelogram if ANY ONE of these is true:

1. **Both pairs of opposite sides parallel**
2. **Both pairs of opposite sides equal**
3. **Both pairs of opposite angles equal**
4. **Diagonals bisect each other**
5. **One pair of opposite sides equal AND parallel**

PART 4: MIDPOINT THEOREM

Theorem 8.8: Midpoint Theorem

The line segment joining midpoints of two sides of a triangle is parallel to the third side and half of it

If E and F are midpoints of AB and AC:

- **EF || BC**
- **EF = ½BC**

Theorem 8.9: Converse of Midpoint Theorem

Line through midpoint of one side parallel to another side bisects the third side

If E is midpoint of AB and EF || BC:

- **F is midpoint of AC**
- **AF = FC**

Applications

Joining Midpoints:

- Midpoints of quadrilateral sides form a parallelogram
- Midpoints of rectangle sides form a rhombus
- Midpoints of rhombus sides form a rectangle
- Midpoints of square sides form a square

QUICK REFERENCE TABLE

Quadrilateral	Opposite Sides	Opposite Angles	Diagonals	All Sides	All Angles
Parallelogram	Equal & Parallel	Equal	Bisect each other	-	-
Rectangle	Equal & Parallel	Equal (90°)	Equal & bisect	-	90°
Rhombus	Equal & Parallel	Equal	Bisect at 90°	Equal	-
Square	Equal & Parallel	Equal (90°)	Equal & bisect at 90°	Equal	90°
Trapezium	One pair parallel	-	-	-	-

PROBLEM-SOLVING FORMULAS

To Prove Parallelogram

Show any ONE:

- AB || DC and AD || BC
- AB = DC and AD = BC
- ∠A = ∠C and ∠B = ∠D

- Diagonals bisect each other

To Prove Rectangle

Show: Parallelogram + (one 90° angle OR equal diagonals)

To Prove Rhombus

Show: Parallelogram + (all sides equal OR diagonals perpendicular)

To Prove Square

Show: Rectangle + all sides equal OR Rhombus + one 90° angle

Using Midpoint Theorem

- To find parallel lines: Join midpoints
- To find lengths: Use $EF = \frac{1}{2}BC$
- To prove bisection: Show line parallel to side through midpoint

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KEY PROOF PATTERNS

Pattern 1: Diagonal Creates Congruent Triangles

In parallelogram ABCD with diagonal AC:

- $\Delta ABC \cong \Delta CDA$ (by ASA using alternate angles)
- Use CPCT to prove sides/angles equal

Pattern 2: Diagonals Bisect Each Other

Mark intersection point O:

- Show $OA = OC$ and $OB = OD$

- Use vertically opposite angles
- Prove triangles congruent

Pattern 3: Adjacent Angles Supplementary

In parallelogram:

- $\angle A + \angle B = 180°$ (co-interior angles)
- Use to find unknown angles

Pattern 4: Midpoint Theorem Application

- Identify midpoints E, F
- Apply EF || third side and EF = ½(third side)
- Use for finding lengths or proving parallelism

COMMON MISTAKES TO AVOID

- Assuming all quadrilaterals are parallelograms
- Forgetting diagonals only bisect in parallelograms (not all quadrilaterals)
- Confusing "equal" diagonals (rectangle) with "perpendicular" diagonals (rhombus)
- Not checking both conditions for special quadrilaterals
- Applying midpoint theorem without checking midpoint condition

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MEMORY AIDS

Parallelogram Properties (DABO):

- **D**iagonals bisect each other

- **A**djacent angles supplementary
- **B**oth pairs opposite sides equal
- **O**pposite angles equal

Special Quadrilaterals:

- **Rectangle** = Right angles
- **Rhombus** = Regular sides (all equal)
- **Square** = Rectangle + Rhombus

Midpoint Theorem: "Joining midpoints gives half and parallel"

ANGLE FORMULAS

In Parallelogram ABCD:

- $\angle A + \angle B = 180°$
- $\angle A = \angle C$
- $\angle B = \angle D$
- $\angle A + \angle B + \angle C + \angle D = 360°$

When Diagonals Meet at O:

In parallelogram: $\angle AOB = \angle COD$ (vertically opposite)
In rhombus: $\angle AOB = 90°$
In square: $\angle AOB = 90°$ and all angles at O equal

This formula sheet covers all essential theorems, properties, and problem-solving techniques for Chapter 8!