Introduction to Euclid's Geometry - Formula Sheet

Basic Definitions

Undefined Terms

- **Point**: Has no part (no dimensions)
- **Line**: Breadthless length (one dimension)
- **Plane Surface**: Has length and breadth only (two dimensions)

Key Concepts

- **Solid**: Has three dimensions (length, breadth, height)
- **Surface**: Boundary of a solid (two dimensions)
- **Straight Line**: A line which lies evenly with points on itself
- **Collinear Points**: Points lying on the same line
- **Non-collinear Points**: Points not lying on the same line

Euclid's Axioms (Common Notions)

Axiom 1: Transitivity of Equality

If $a = b$ and $b = c$, then $a = c$ "*Things which are equal to the same thing are equal to one another*"

Axiom 2: Addition Property

If $a = b$, then $a + c = b + c$ "*If equals are added to equals, the wholes are equal*"

Axiom 3: Subtraction Property

If $a = b$, then $a - c = b - c$ "*If equals are subtracted from equals, the remainders are equal*"

Axiom 4: Superposition Principle

If two things coincide with one another, they are equal "*Things which coincide with one another are equal to one another*"

Axiom 5: Whole-Part Relationship

The whole is greater than the part **If A = B + C, then A > B and A > C**

Axiom 6: Double of Same Things

Things which are double of the same things are equal to one another

Axiom 7: Halves of Same Things

Things which are halves of the same things are equal to one another

Euclid's Postulates

Postulate 1: Line Through Two Points

A straight line may be drawn from any one point to any other point

Modern Form: Through any two distinct points, there passes one and only one straight line

Postulate 2: Extension of Line Segment

A terminated line can be produced indefinitely

Modern Form: A line segment can be extended on either side to form a line

Postulate 3: Circle Construction

A circle can be drawn with any centre and any radius

Postulate 4: Right Angles Equality

All right angles are equal to one another

Modern Form: All right angles are equal (= 90°)

Postulate 5: Parallel Lines (Parallel Postulate)

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Modern Form: If $\angle 1 + \angle 2 < 180°$, then lines will meet on that side

Additional Axioms (Modern)

Axiom 5.1: Unique Line

Given two distinct points, there is a unique line that passes through them

Key Theorems and Results

Theorem 5.1: Intersection of Lines

Two distinct lines cannot have more than one point in common

Proof Method: Contradiction (assume two points, leads to violation of unique line axiom)

Important Formulas and Relationships

Line Segments

- **AB + BC = AC** (when B lies between A and C)
- **If C is midpoint of AB**: $AC = BC = \frac{1}{2}AB$

Equality Properties

- **Reflexive**: a = a
- **Symmetric**: If a = b, then b = a
- **Transitive**: If a = b and b = c, then a = c

Construction Principles

- **Equilateral Triangle**: Can be constructed on any given line segment
- **Circle**: Determined by center and radius
- **Line**: Determined by any two points on it

Proof Techniques

Direct Proof

1. Start with given information
2. Use axioms and postulates
3. Apply logical reasoning
4. Reach the conclusion

Proof by Contradiction

1. Assume the opposite of what you want to prove
2. Show this leads to a contradiction with known axioms/postulates
3. Conclude the original statement must be true

Key Geometric Relationships

Collinearity

- **Points A, B, C are collinear** ⟺ They lie on the same straight line
- **AB + BC = AC** (B between A and C)

Congruence

- **Figures are congruent** ⟺ They have same shape and size
- **Corresponding parts** of congruent figures are equal

Superposition

- If one figure can be placed exactly over another, they are congruent
- Based on Axiom 4: "Things which coincide are equal"

Problem-Solving Framework

Step 1: Identify Given Information

- What is provided in the problem
- What relationships are established

Step 2: Identify What to Prove

- Clear statement of the conclusion
- What needs to be established

Step 3: Choose Method

- Direct proof using axioms/postulates
- Contradiction method

- Construction method

Step 4: Apply Logical Steps

- Use relevant axioms and postulates
- Maintain logical sequence
- State reasons for each step

Step 5: State Conclusion

- Clear statement of what has been proved
- Reference to the required result

Memory Aids

The "SUPER" Axioms

- **S**ame things equal (Axiom 1)
- **U**nion preserves equality (Axiom 2)
- **P**artition preserves equality (Axiom 3)
- **E**xact overlap means equal (Axiom 4)
- **R**emainder less than whole (Axiom 5)

Postulate Sequence

1. **Connect** any two points
2. **Extend** any line segment
3. **Circle** from any center
4. **Right** angles all equal
5. **Parallel** postulate (most complex)

Common Applications

Proving Congruence

- Use superposition principle (Axiom 4)
- Apply construction postulates
- Utilize equality axioms

Establishing Relationships

- Whole-part relationships (Axiom 5)
- Equality chains (Axiom 1)
- Addition/subtraction properties (Axioms 2, 3)

Geometric Constructions

- Basic constructions using Postulates 1-3
- Equilateral triangles
- Perpendicular bisectors
- Angle bisectors

Note: This chapter establishes the logical foundation for all geometric proofs. Master these axioms and postulates as they form the basis for advanced geometry.